Approach of the Attack Analysis to Reduce Omissions in the Risk Management

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Abstract

The article is dedicated the attack analysis and reducing mistakes and miscalculations in risk management. Widespread use of game theory in the analysis of attacks on information resources and countering them can significantly reduce errors and miscalculations that occur in risk management, which in turn minimizes the negative and adverse political, social, and financial consequences for the subjects of information warfare. The solution to the problems of information confrontation is impossible without the development of new theoretical and methodological principles for the analysis of confrontation processes. The authors have offered and studied the scheme of finding sustainable strategies, which ensure the neutralization of the enemy. The scheme for finding sustainable strategies always turns out to be useful in many problems.

Keywords

Cyberspace, risk management, sustainable strategy, cyberwar, hybrid war, game theory, payoff function, counteraction, neutralization, attack on information, conflict management.

1. Introduction

In recent years, due to the rapid development of operations research of systems engineering in solving risk management problems, it has become possible to study conflict situations taking into account situations of uncertainty.

The theoretical basis of risk management in conflict situations is game theory. Hybrid war and cyber warfare contributed to the widespread adoption of game theory [1]. New forms and methods of counteraction have appeared. The classic forms of confrontation have been replaced by hybrid methods. They are of a hidden nature and are carried out mainly in the political, economic, informational, and other spheres. Solving the issues of risk management and information protection, countering attacks, and information impacts remain relevant for all of us.

Nowadays, game-theoretic methods [2] are successfully used to solve a wide variety of issues. The application of game theory in solving problems of risk estimating in information wars, information and information-psychological confrontation, information and geopolitical areas gives especially great benefit.

Game theory is a mathematical theory of conflict situations. In these situations, the interests of two or more parties collide, which pursue different, opposite goals. The direct subject of study of the game theory is the mathematical analysis of a formalized model of conflict, which takes into account the peculiarities of a real conflict situation. The technique itself is the formalization of a specific conflict situation does not apply to the mathematical theory of games. It is within the competence of specialists in the field, which is affected by this conflict situation.

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Each conflict situation, in terms of risk assessment, is a complex situation. Risk analysis is hampered by many secondary factors. Therefore, in order to make possible a mathematical analysis of the situation, it is necessary to abstract from random factors and builds a simplified formalized model of the process and risk management factors. In this case, the formalization should be such that the possible ways of behavior of the participants and the results are visible, to which all possible combinations of actions of all participants in the conflict lead.

2. Literature Survey

Following modern research trends can be identified in this field: building influence models (information cascades (IC) [3]; linear thresholds (LT) [4], probabilistic models [5]; construction of effective algorithms for maximizing the impact (based on the apparatus of submodular functions (greedy algorithm) and its improvement, CELF [6], CELF ++ [7]); using local properties of the graph (LDAG [8], SimPath [9]); thinning the graph [10]; simulated annealing [11]; network monitoring optimization algorithms [6]; variations of the influence maximization problem and solution algorithms (maximizing influence blocking [12], maximizing influence taking into account time [13], thematic distribution of influence [14]); game-theoretic models of information influence [15, 16].

3. Purpose and Objectives

The analysis of scientific and technical literature [17–21] showed that to date the following issues of game theory application have not been solved within the problem of risk management for information protection:

- The task of risk management for information protection has not been structured.
- No areas of risk quantitative estimates have been found.
- No guaranteed assessments of the risk level of information security were found.
- Optimal strategies for attacking and protecting information have not been found.
- The solution of information protection issues described by stochastic models is not fully found;
- The behavior of information attacks during information confrontations has not been studied.

Modeling of information attack processes involves the risk management and reflection in the developed models of dynamic properties due to the conflicting nature and related ideas about the optimal distribution of information resources of players [22].

Mathematical modeling of physical processes by methods of game theory is based on the following factors that verbally determine the essence of this theory [23]:

- The presence of a system of differential equations, which describes the change over time in the parameters of the processes being modeled.
- Definition of admissible controls of players, in the form of a class of functions on which the corresponding restrictions are imposed.
- Goals of players in the form of functionalities.
- Information that is available to players at the beginning of the game and in the process.

Thus, the use of game theory in information warfare for the purpose of risk management requires detailed research, which is the purpose of this article.

4. Solutions of Games with a Choice of Time

Tasks related to the timing of actions occur in many problems of information confrontation, which use game theory applications [24]. In such situations, the possible actions of the players are set in advance. During the action, the goal is set by strategic decisions of the players (the attacker and the defending side). In general, the payoff function of such games has the following form [25]

$$M(x,y) = \begin{cases} K(x,y)\ for\ x < y, \\ I(x)\ for\ x = y, \\ L(x,y)\ forx > y \end{cases} \tag{1}$$

here various restrictions can be imposed on the functions K, I, and L. They are determined by the specific conditions of the problem being solved.

Many kinds of research [26, 27] have been devoted to the study of games with payoff function (1). The corresponding mutually exclusive classification of all types of games are given in Karlin's monograph [27]. Before starting the results, we introduce some notation. We denote the distribution function *P(x)*, which has a jump in α at zero and a jump in β at unity, by $P(x) = (\alpha I_0, P_{ab}(x), \beta I_1)$ where the distribution density $P_{ab}(x)$ is a continuous function in the entire interval $[a, b] \subset [0,1]$.

Therefore, the following theorem is true.

Theorem 1 [27]. Let the payoff function of a continuous game has the following form:

$$M(x,y) = \begin{cases} K(x,y)\ for\ x < y, \\ L(x,y)\ for\ x > y, \\ K(x,y) = L(x,x) \end{cases} \tag{2}$$

The functions K and L satisfy the following conditions:

1. The functions $K(x,y)$ and $L(x,y)$ have continuous third partial derivatives in their domains of definition.

2. The derivatives $K_{xx}(x,y)$ and $K_{yy}(x,y)$ are strictly negative for $x \le y$, and the derivatives $L_{xx}(x,y)$ and $L_{yy}(x,y)$ are strictly negative for $x \ge y$.

3. The function $K(x,y)$ strictly increases in *y* and strictly decreases in *x*, and the function $L(x,y)$ strictly increases in *x* and strictly decreases in *y*.

Then both sides have unique optimal mixed strategies of the following form

$$F(x) = (\propto I_0, f(x), \beta I_1), \tag{3}$$

$$H(y) = (\gamma I_0, h(y), \delta I_1). \tag{4}$$

Here, the function $f(x)$ and $h(y)$ are continuous in the entire interval [0,1] and are obtained as the only solutions of a pair of integral equations:

$$\propto p_1 + \beta p_2 = f + T_f, \tag{5}$$

$$\gamma p_1 + \delta p_2 = h + R_h \tag{6}$$

$$T_f = \int_0^y \frac{K_{yy}(x,y)}{K_y(y,y) - L_y(y,y)} f(x)dx + \int_y^1 \frac{L_{yy}(x,y)}{K_y(y,y) - L_y(y,y)} f(x)dx \tag{7}$$

$$R_h = \int_0^x \frac{L_{xx}(x,y)}{L_x(x,x) - K_x(x,x)} h(y)dy + \int_x^1 \frac{K_{xx}(x,y)}{L_x(x,x) - K_x(x,x)} h(y)dy \tag{8}$$

$$\begin{aligned} p_1 &= -\frac{K_{yy}(0,y)}{K_y(y,y) - L_y(y,y)} \\ p_2 &= -\frac{L_{yy}(1,y)}{K_y(y,y) - L_y(y,y)} \end{aligned} \tag{9}$$

$$q_1 = -\frac{L_{xx}(x,0)}{L_x(x,x) - K_x(x,x)} \tag{10}$$

$$q_2 = -\frac{K_{xx}(x,1)}{L_x(x,x) - K_x(x,x)}$$

The constants $\propto, \beta, \gamma, \delta$ are determined from the following conditions:

$$\int_0^1 f(x)dx = 1 - \propto - \beta, (0 \leq \propto, \beta \leq 1) \tag{11}$$

$$\int_0^1 h(y)dy = 1 - \gamma - \delta, (0 \leq \gamma, \delta \leq 1) \tag{12}$$

Thus, the solution of the game under consideration is reduced to the solution of integral equations. This solution is a simple task. These equations are classic integral equations. In particular, we use the expansion of unknown functions f and h in a Neumann series in order to find analytical solutions.

There are general results that can be formulated as the following theorem [27].

Theorem 2. Let the payoff function of a continuous game has the following form:

$$M(x,y) = \begin{cases} K(x,y)\ for x < y, \\ l(x)\ for\ x = y, \\ L(x,y)\ for\ x > y \end{cases} \tag{13}$$

The functions K, l, L satisfy the following conditions:

1. The functions $K(x,y)$ and $L(x,y)$ are defined and have continuous second partial derivatives on closed triangles $0 \leq x \leq y \leq 1$ and $0 \leq y \leq x \leq 1$, respectively.
2. The $l(1)$ value lies between $K(1,1)$ and $L(1,1)$; the $l(0)$ value lies between $K(0,0)$ and $L(0,0)$.
3. $K_x(x,y) > 0$ and $L_x(x,y) > 0$ are located in the corresponding closed triangles with the possible exception of $L_x(1,1) = 0$; $K_y(x,y) < 0$ and $L_y(x,y) < 0$ in the corresponding closed triangles with the possible exception of $K_y(1,1) = 0$.

Then, both sides have optimal strategies of the following form

$$F(x) = \left(\propto I_0, f_{\propto_1}, \beta I_1\right),$$

$$H(y) = \left(\gamma I_0, h_{\propto_1}, \delta I_1\right),$$

The distribution densities $f_{\propto_1}$ and $h_{\propto_1}$ are determined as solutions of the following integral equations:

$$f_{a_1}(t) - \int_a^1 T_{a_1}(x,t) f_{\propto_1}(x)dx = \propto p_1(t) + \beta p_2(t); \tag{14}$$

$$h_{a_1}(u) - \int_a^1 U_{a_1}(u,y) h_{\propto_1}(y)dy = \gamma q_1(u) + \delta q_2(u); \tag{15}$$

$$T_{a_1}(x,t) = \begin{cases} \dfrac{-K_y(x,t)}{K(t,t) - L(t,t)};\ a \leq x < t \leq 1; \\ \dfrac{-L_y(x,t)}{K(t,t) - L(t,t)};\ \mathrm{a} \leq t \leq x \leq 1; \end{cases} \tag{16}$$

$$U_{a_1}(u,y) = \begin{cases} \dfrac{L_x(u,y)}{K(u,u) - L(u,u)};\ a \le y < u \le 1; \\ \dfrac{K_x(u,y)}{K(u,u) - L(u,u)};\ a \le u \le y \le 1; \end{cases} \tag{17}$$

$$p_1(t) = \frac{-K_y(0,t)}{K(t,t) - L(t,t)}$$
$$p_2(t) = \frac{-L_y(1,t)}{K(t,t) - L(t,t)} \tag{18}$$

$$q_1(u) = \frac{L_x(u,0)}{K(u,u) - L(u,u)}$$
$$q_2(u) = \frac{K_x(u,1)}{U(u,u) - L(u,u)} \tag{19}$$

The constants $\propto, \beta, \gamma, \delta$ and a are determined from the following conditions

$$\int_a^1 f_{a_1}(x)dx = 1 - \propto - \beta, (0 \le \propto, \beta \le 1) \tag{20}$$

$$\int_a^1 h_{a_1}(y)dy = 1 - \gamma - \delta, (0 \le \gamma, \delta \le 1) \tag{21}$$

Remark 1. It follows from the equation (13) that if $K(1,1) < L(1,1)$, then the point $x = 1$ and $y = 1$ is a saddle point for $M(x,y)$. This follows from condition (2) of Theorem 1.

Corollary 1. For the case of $l(x) = 0$ and $-K(x,y) = L(x,y)$, the game is called symmetric.

The symmetric game is investigated for the case when the function $M(x,y)$ in the region 0$\le$ $(x \le y \le 1)$ is continuous in both variables and has continuous first-order partial derivatives $M_x(x,y) \ge 0, M_y(x,y) \le 0$ for $x \le y$ and the set of points for which $M_x(x,y) = 0$ or $M_y(x,y) = 0$ does not contain any interval of the form $x = const$, $\beta_1 < y < \beta_2$ or the form $y = const$, $\propto_1 < x < \propto_2$.

The optimal strategy is unique and has the following form for $K(1,1,) \le 0$

$$F(x) = I_1 = \begin{cases} 0\ for\ 0 \le x < 1, \\ 1\ for\ x = 1. \end{cases} \tag{22}$$

There is an optimal strategy of the following form for $K(0,1,) > 0$:

$$F(x) = I_0 = \begin{cases} 0\ for\ x = 0, \\ 1\ for\ 0 < x \le 1. \end{cases} \tag{23}$$

In the case $K(0,1,) < 0 < K(1,1)$, we can assume without loss of generality $K(x,x,) > 0$ for $0 < x \le 1$. Then there is a uniquely defined interval of the form $[a,1], 0 \le a \le 1$, such that the optimal strategy is as follows:

$$F(x) = \begin{cases} 0 \, for \, x = 0, \\ \propto for \, 0 < x \leq a, \\ \propto - \int_a^x f_{a_1}(z)dz \, for \, a < x \leq 1 \end{cases} \quad (24)$$

The function $f_{a_1}(x)$ is a continuous, positive function. The parameter $\propto$ is the jump of $F(x)$ at zero and is determined from the normalization equation:

$$\int_a^1 f_{a_1}(z)dz = 1-\propto \quad (25)$$

From Theorem 1 it follows that the optimal strategy $F(x)$ for a symmetric game in the case under consideration exists only if it is possible to find numbers $a, \propto$, that satisfy the conditions $0 \leq a, \propto < 1$ and such a continuous non-negative function $f_{a_1}(x)$ for $a < x < 1$ such that

$$aK(0,y) + \int_a^y K(x,y)f_{a_1}(x)dx - \int_y^1 K(y,x)f_{a_1}(x)dx = 0, (a < y < 1) \quad (26)$$

Remark 2. The case of the function $M(x,y)$, which increases in y and decreases in x, using the substitution $z = 1 - x, \eta = 1 - y$ reduces to the case of increasing in x and decreasing in y, which was considered in the Theorem 1.

Remark 3.

If in the Theorem 1, instead of the condition (1), we assume that $\left(K_y(y,y) - L_y(y,y)\right) > 0$ and $\left(K_x(x,x) - L_x(x,x)\right) > 0$, then one can verify [27, 28] that the optimal strategies of both parties have the form of the distribution function $F(x) = (\propto I_a, f_{ab}(x), \beta I_b)$ and $H(y) = (\gamma I_a, h_{ab}(y), \delta I_b)$, where $\propto, \beta, \gamma, \delta \geq 0$, and the function $f_{ab}(x)$ and $h_{ab}(y)$ are obtained in the form of Neumann series in the eigenfunctions of the conjugate integral equations

$$f_{ab}(t) - \int_a^b T_{ab}(x,t)f_{ab}(x)dx = \propto p_1(t) + \beta p_2(t) \quad (27)$$

$$h_{ab}(t) - \int_a^b U_{ab}(u,y)h_{ab}(y)dy = \gamma q_1(u) + \delta q_2(u) \quad (28)$$

Further, consider a special class of symmetric games for which $M(x,y)$ is not necessarily continuous in the set of variables at the points (0,0) and (1,1), and it is only required that the following limits exist

$$K(0{,}0) = \lim_{y \to 0} K(0,y) \, ; K(1{,}1) = \lim_{x \to 0} K(x,1). \quad (29)$$

We will assume that

$$K(x,y) = k\left(\frac{x}{y}\right), \quad (30)$$

The function $k(u)$ is continuously differentiable in the interval $0 \leq u \leq 1$, and its derivative $k'(u)$ does not change the sign on this interval. Moreover, the set of points u for which $k'(u) = 0$ does not contain any interval.

It is easy to see that for the equation $k'(u) \geq 0$, the negative strategy is $F(x) = I_1$, for the equation $k(1) \leq 0$ and $F(x) = I_0$, for $k(1) \geq 0$. The proof of this fact is based on the idea of finding sustainable strategies. For this, we write the equality

$$C_1(F, +0) = C_1(F, 0) + \propto K(0,0) = C_1(F, 0) + \propto k(0). \tag{31}$$

The validity of this equality is established using (29). Indeed, for the equation $\delta > 0$ we have the following expression

$$C_1(F, \delta) = \int_0^{\delta-0} K(x, \delta) dF(x) - \int_\delta^1 K(\delta, x) dF(x), \tag{32}$$

$$C_1(F, 0) = -\int_{+\delta}^1 K(0, x) dF(x). \tag{33}$$

Thus

$$C_1(F, \delta) - C_1(F, 0) =$$
$$= \propto K(0, \delta) + \int_{+0}^{\delta-0} K(x, \delta) dF(x) - \int_\delta^1 K(\delta, x) dF(x) + \int_{+\delta}^1 K(0, x) dF(x) \tag{34}$$

The first term on the right-hand side of formula (34) as $\delta \to 0$, taking into account (29), tends to $\propto K(0,0)$. In order to estimate the integrals in (34), for a given $\varepsilon > 0$, we choose η such that the total variation of $F(x)$ in $[0, \eta]$ is less than $\varepsilon/4K_0$, where $K_0 = sup|K(x, y)|$. Then the first integral will be less than $\varepsilon/4$, and the next two can be represented as:

$$\int_{1+0}^{\eta} K(0, x) dF(x) - \int_\delta^\eta K(\delta, x) dF(x) +$$
$$+\int_\eta^{1+0} \big(K(0, x) - K(\delta, x)\big) dF(x) = I_1 + I_2 + I_3. \tag{35}$$

It is obvious from (35) that all $|I_i| \leq \varepsilon/4, i = 1,2,3$. Hence, this proves the validity of (31).

Let us first take the value $a = 0$. From $C_1(F, y) = 0$ for $a < y < 1$ it follows that $C_1(F, +0) = 0$. For $a > 0$, it should be $C_1(F, 0) = 0$. It leads to a contradiction with (31), due to the expression $k(0) < 0$. On the other hand, for $\propto = 0$ we have the following expression

$$C_1(F, +0) = -\int_0^1 k(0) f(x) dx = -k(0) \int_0^1 f(x) dx = -k(0) > 0 \tag{36}$$

that it is also impossible. If we take $\propto > 0$, then from $C_1(F, a) = 0$, and strict decrease of the function we obtain

$$C_1(F,y) = \propto k(0) - \int_a^1 k\left(\frac{y}{x}\right) f(x)dx \tag{37}$$

on the interval $0 < y \leq \propto$ we get $C_1(F,+0) > 0$. If $\propto > 0$ and $C_1(F,0) = 0$, then from expression (31) we obtain $C_1(F,+0) = \propto k(0) < 0$. This is a contradiction. Hence $\propto > 0$ and $\propto = 0$. In this case, expression (26) is equivalent to the expression $C_1(F,y) = 0$ on the interval $(\propto, 1)$ under the condition $C_1'(F,y) = 0$. It follows from this expression that, we obtain an integral equation for determining the density $f(x)$

$$2k(1)f(y) = \int_a^y \frac{x}{y^2} k'\left(\frac{x}{y}\right) f(x)dx + \int_y^1 \frac{f(x)}{x} k'\left(\frac{y}{x}\right) dx, (a < y < 1). \tag{38}$$

In this case, the normalization condition must be satisfied

$$\int_a^1 f(x)dx = 1.$$

The Remark 4.

For the case of $k'(u) \leq 0$, it can be shown [28, 29] that optimal strategies are $F(x) = \propto I_0 + \beta I_1$. In addition, it is easy to check the validity of the following expressions

$$F(x) = \begin{cases} I_1(x) for\ k(0) < 0, \\ \propto I_0(x) + (1-\propto) I_1(x)\ for\ k(0) = 0\ (0 \leq \propto \leq 1), \\ I_0(x)\ for\ k(0) > 0. \end{cases}$$

The solution of the game $G(M, [0,1])$ with the payoff function $M(x,y), (0 \leq x, y \leq 1)$ is called a pair of distribution functions (strategies) F_1^* and F_2^* and a real number υ (value of the game) that satisfies the condition

$$\int_0^1 M(x,y)d\,F_2^*(y) \leq \upsilon \leq \int_0^1 M(x,y)d\,F_1^*(x), 0 \leq x, y \leq 1.$$

It follows from this expression that if the player $G1$ uses the strategy F_1^*, then the average payoff is calculated by the following formula

$$F(F_1^*, F_2) = \iint_0^1 M(x,y)dF_1^*(x)dF_2^*(y).$$

This payoff cannot be less than the number υ, i.e. the player $G1$, as it were, neutralizes the opponent's actions. And, conversely, if the player $G2$ applies the strategy F_2^*, then his average loss $F(F_1, F_2^*)$ will always be greater than the number υ, regardless of the actions of the player $G1$. Therefore, it is natural that each player should strive to choose such distribution functions F_1^* and F_2^*, which could neutralize the opponent's actions. Indeed, for the player $G1$, the best strategy is a strategy that makes his average winnings as large as possible within reason, regardless of the opponent's actions. Moreover, conversely, the player $G2$ must choose a strategy that would provide him, within reasonable limits, the smallest possible loss, regardless of the actions of the player$G1$. Naturally, if the game has an equilibrium position on the space of distribution functions, then only in this case the players can choose optimal strategies [30].

In general, the player $G1$ can guarantee himself a payoff of at least

$$\upsilon_1 = \max_{F_1} \min_{y} \int_0^1 C_1(F_1) d\, F_2(y) = \max_{F_1} \min_{y} C_1[F_1(y)]. \quad (39)$$

Here

$$C_1(F_1) = \int_0^1 M(x, y) dF_1(x).$$

Similarly, the player $G2$, by the appropriate choice of the distribution function $F_2(y)$, can guarantee himself a loss of no more than

$$\upsilon_2 = \min_{F_2} \max_{F_1} \int_0^1 C_2(F_2) d\, F_1(x) = \min_{F_2} \max_{x} C_2[F_2(x)]. \quad (40)$$

Here

$$C_2(F_2) = \int_0^1 M(x, y) dF_2(y).$$

From equations (39) and (40) we obtain

$$\begin{aligned} \upsilon_1 &\geq \min_{y} C_1(F_1) \\ \upsilon_2 &\leq \max_{x} C_2(F_2), \end{aligned} \quad (41)$$

Let the player $G2$ choose the distribution function $F_{20}(y)$ as his strategy, and let the player $G1$ know this choice. Naturally, assuming such an opportunity, the $G2$ player should strive to find a sustainable strategy. It follows from (41), it becomes clear that if the value $C_2(F_2)$ has a maximum, then the player $G1$ will always get the best result, choosing a point χ_0 that corresponds to this maximum

$$\upsilon_0 \leq C_2\big(F_2(\chi_0)\big) = \max_{\chi} C_2(F_2(\chi)).$$

It would be beneficial for the player $G2$ to bring the value of $C_2(F_2(x))$ to a minimum, but this is not always possible. The player cannot influence the form of the payoff function and the choice of χ_0 by the player $G1$. Nevertheless, the player $G2$ can, in any case, try to choose the strategy $F_{20}(y)$ so that the value of $C_2(F_2)$ does not have a single maximum, that is, so that its "curve" has a flat top.

Similarly, if the player $G2$ has learned the strategy of the player $G1$, then he will always choose the point y_0 at which the function $C_1(F_1(y))$ will take the minimum value. In this case, the task of the player $G1$ is to choose such a strategy $F_{10}(x)$ so that the function $C_1(F_1(y))$ does not have a single minimum.

We denote $\Omega_1 = \{x: C_2(F_2(x)) = \nu_1 = const\}$ and $\Omega_2 = \{y: C_1(F_1(y)) = \nu_2 = const\}$, where ν_1 and ν_2 are arbitrary numbers, and $\upsilon_1 \leq \nu_1 \leq \nu_2 \leq \upsilon_2$.

If there is such a pair of real numbers $(\nu_1 \leq \nu_2)$ and a pair of distribution functions (F_1, F_2), which simultaneously satisfies the following conditions

$$C_1(F_1(y)) \begin{cases} = \nu_1 \; for \; y \in \Omega_2, \\ > \nu_1 \; for \; y \notin \Omega_2. \end{cases} \tag{42}$$

$$C_2(F_2(x)) \begin{cases} = \nu_2 \; for \; x \in \Omega_1, \\ > \nu_2 \; for \; x \notin \Omega_1, \end{cases} \tag{43}$$

then the functions F_1 and F_2 will be called stable [27, 30] strategies.

The question of the existence of sustainable strategies for the payoff function $M(x, y)$ in most cases remains unsolved. The scheme itself finding sustainable strategies is always useful in many applications and, in particular, in the game theory with a choice of a moment in time. Such games do not require the definition of strategies that neutralize the enemy. It turns out [27, 30] that instead of them one can be content with partially stable strategies, i.e. strategies that provide the player with a stable position in a certain subinterval of the unit interval.

5. Conclusion

Widespread use of game theory in the analysis of attacks on information resources and countering them can significantly reduce errors and miscalculations that occur in the risk management of information security, which in turn minimizes the negative and adverse political, social, and financial consequences for the subjects of information warfare.

Systematic studies of the behavior for complex dynamic processes require consideration of a large number of risks, features, and relationships of typical attacks on information and informational influences. The investigated features contradict one another; however, each of them cannot be neglected, since they give us a complete picture of the process that is investigated or simulated.

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