Special Issue: Socio-political and humanitarian changes in the context of society's transformation

The role of interactive learning in mathematics education: Fostering student engagement and interest

Liudmyla Hetmanenko[a] ✉

[a]Department of Natural Sciences and Mathematics Education and Technologies, Institute of In-Service Teacher's Training, Borys Grinchenko Kyiv Metropolitan University, Kyiv, Ukraine.

Abstract This article examines the role of active learning in mathematics and its impact on stimulating students' interest in the subject. It analyses contemporary methods of active learning, such as group projects, discussions, and problem-based tasks, and considers their applicability in teaching mathematics. The article discusses different methods of stimulating students' interest in mathematics, including the use of practical examples, interactive teaching methods, and support for independent research. Based on the analysis conducted, recommendations are provided for creating a learning environment that fosters the maximum development of students' interest in mathematics and enhances the effectiveness of their learning. The article highlights the significance of contemporary teaching methods in mathematics. These methods concentrate on fundamental mathematical concepts and foster critical thinking, communication skills, and independent research skills. The article discusses the importance of studying formal geometry in mathematical education and proposes original methods for solving classical and authorial problems based on new dependencies that were previously unknown. In conclusion, the article highlights that active learning is a crucial component of successful mathematics education. It promotes not only the assimilation of material but also the development of skills necessary for successful adaptation and application of knowledge in various life spheres and careers.

Keywords: active learning, mathematics, interest in the subject, educational process, stimulation of interest, critical thinking

1. Introduction

Mathematics is often seen as a subject that requires a high degree of abstraction and careful memorization of formulas. However, modern approaches to mathematics education increasingly recognize the importance of actively engaging students in the learning process. Active learning is a method that emphasizes the role of students as active participants rather than passive recipients of information. This article explores the significance of active learning in mathematics, its impact on understanding mathematical concepts, developing critical thinking, and stimulating student interest in the subject. We will also analyse the importance of student feedback in the learning process and ways to stimulate their active participation and creative thinking in the learning process through solving problems with original solution methods and authorial tasks built on new dependencies previously unknown (Robinson & Elinbee, 2022).

Although mathematics plays a crucial role in modern education and scientific research, many students struggle to grasp this subject and often lose interest in it. One of the main challenges facing educational institutions and mathematics teachers is that traditional teaching methods, which rely on mechanical memorization of formulas, are not sufficiently effective in stimulating students' interest and comprehension of the material. It is important to note that students may feel disconnected from mathematics due to its abstract nature and apparent detachment from real life. This disconnection can lead to a decline in performance, reduced motivation, and even a lack of interest in studying mathematics (Mamontov, 1989). The issue is that conventional methods of teaching mathematics are not sufficiently tailored to meet the requirements of contemporary students. They do not encourage the development of critical thinking, creative problem-solving abilities, or independent exploration of mathematical concepts. There is a requirement for the creation and execution of active learning techniques that effectively engage students with mathematics. These techniques should stimulate their interest and develop not only mathematical skills but also critical thinking, communication skills, and independent research based on authorial tasks built on new dependencies previously unknown (Burns & Ostfeld, 2020).

This article is aimed at considering the role of active learning in mathematics, considering students' comments on the learning process and stimulating their interest in the subject through the author's tasks based on new dependencies not known before.

Objectives of the article:

1. To analyse modern methods of active learning, such as group projects, discussions, problem-based tasks and others, and consider their applicability in the context of mathematics.
2. Based on the analysis, develop five author's tasks and propose their solution using original methods based on new dependencies not yet known.
3. To consider various strategies and methods that can be used to stimulate students' interest in mathematics, including the use of practical examples and support for independent research

2. Research methods

The following research methods are used to achieve the goal and solve the tasks of the article:

— *Literature review.* An extensive analysis of existing scientific articles, studies, books and other publications on active learning in the context of mathematics. This method provides an overview of current trends, methods and research results in this area.
— *Analysis of pedagogical practice.* The study of the experience of mathematics teachers who use active learning methods in their practice. This includes classroom observation, analysis of teaching materials, interviews with teachers, and observation of student reactions.
— *Modelling the educational process.* Mathematical models can be developed to describe and analyse various aspects of the educational process, including the interaction between teachers and students, the dynamics of knowledge acquisition and the formation of student motivation.

The combination of these methods allows us to get a comprehensive view of the role of active learning in learning mathematics, as well as to offer recommendations for optimizing the educational process and improving the effectiveness of student learning in this area.

3. Literature review

Many authors have analysed and published various manuals that offer different types of mathematics problems, including geometry, and their solutions. It is important to distinguish among them the work of I.A. Kushnir (1991), R.V. Skiba (2018), Yu.I. Mamontov, S.I. Sydorenko (1988; 1989), I.N. Peterson (1999), T.H. Semenova, A.M. Shvab (2018), T.M. Kovalchuk, O.H. Maksymiuk (2017), V.K. Petrov (2016), O.V. Sokolov, M.S. Yashchenko (2018), O.P. Khimich, V.V. Chornobai (2017), A.I. Kostrykin, Yu.Y. Manyn (1986), A.H. Kostriuk, N.A. Kramer (1981), B.A. Dubrovin, B.A. Novikov, A.T. Fomenko (1986), P. Ly (1980), S.V. Fomin, Y.R. Shafarevych (1986), J. W. Milnor (1984), I.V. Serhiienko (1988), V.V. Holubiev and V.S. Kondratenko (1983), R. C. Hartshorne (1999), R.V. Skiba (2018).

M. Burns and D. Ostfeld (2020) discuss active learning strategies for teaching mathematics and provide evidence of their effectiveness in improving student performance. M. Frank, D. Maxwell, and G. Carbon (2021) describe practical methods for implementing these strategies in the classroom, including examples of sessions and tips for teachers. D.A. Robinson and D. M. Elinbee (2022) conducted research that demonstrates how active learning can improve student performance and engagement in college mathematics courses. V. Davis, D. Reddy, and K. Lowy (2020) analysed different pedagogical approaches to active learning in higher mathematical education, discussing their pros and cons. L. Herndon and S. Weiss (2021) propose strategies for creating interactive mathematics classes that promote active student participation. J. Johnson (2020) reviews studies demonstrating the effectiveness of active learning methods in improving student performance in mathematics. D. Martin and S. Griffin (2022) describe active learning methods that can develop critical thinking in students studying mathematics. D. Tracy and S. Shellenberg (2021) discuss active learning approaches to improve student performance in mathematics. D. Morris and S. Fitzgerald (2020) offer strategies to develop mathematical thinking in students. J. Levin and J. Wilson's (2022) research demonstrates that active learning leads to higher student performance and greater engagement in mathematics courses.

4. Results and discussions

Mathematics, with its rigorous logic and abstract concepts, often causes students to feel anxious and even alienated. However, modern teaching methods based on the active involvement of students in the learning process can significantly improve their attitude towards this subject.

Below are some of the benefits of active learning in mathematics:

— *Stimulation of thinking activity: Active learning, such as group projects, discussions, and problem-based tasks, stimulate students to think. This is especially important in mathematics, as this subject requires the development of logical thinking and the ability to apply knowledge in practice.*

- *Increasing understanding of concepts: Engaging students in active activities such as problem-solving, discussion, and research helps them to gain a broader understanding of mathematical concepts. When students explore the material themselves and share ideas, they are more likely to remember and retain information better.*
- *Development of communication skills: Active learning promotes communication skills, such as explaining one's thoughts and beliefs and listening and considering the opinions of others. This is particularly important in mathematics, as it often requires discussion and explanation* of mathematical ideas.

Stimulating students' interest in mathematics requires a creative approach and a constant striving for innovation in the educational process. Creating an engaging and stimulating learning environment will help to attract students' attention and motivate them to actively participate and successfully master mathematical knowledge.

Stimulating students' interest is formed by the following components.

1. *The use of practical examples.* Using real-life examples helps students see the practical application of mathematics in their lives. This can inspire students and show them that mathematics is not only an abstract theory, but also a tool for solving real problems.
2. *Support for independent research.* Giving students the opportunity to research topics of interest on their own and develop their mathematical skills can greatly increase their motivation and interest in the subject.
3. *Feedback and assessment.* It is important to provide students with feedback on their progress and achievements, as well as support in case of difficulties. In addition, students' assessment should be based not only on the result, but also on their participation in active learning methods.
4. *Support for cross-curricular connections.* Integrating mathematics with other subjects and real-world situations helps students better understand the importance and applicability of mathematical concepts in different areas of life and learn to solve interdisciplinary problems.

Successful implementation of active learning in mathematical education requires effective teaching methods and support from educational institutions and teachers. Active learning in mathematics helps students grasp the concepts of this complex subject and develops skills that will be useful to them in their future lives and careers.

Various concepts in geometry have been studied since ancient times, such as the radius of the circumscribed circle around a triangle, the inscribed circle within a triangle, externally tangent circles touching one side of the triangle and extending two others, the radius of the Euler circle passing through the midpoints of the triangle's sides, its altitudes' bases, and the midpoints of segments connecting the triangle's vertices to the orthocenter. The Apollonian circle is a circle that is described around a right triangle, with its legs being the bisectors of the adjacent angles at the triangle's vertex. The semicircle inscribed within a triangle, touching two sides while its diameter belongs to the third side, has helped to create new problems and formulas, justify and prove properties of the semicircle, and solve previously known problems using new methods.

A systematic analysis of scientific sources was conducted to determine the availability of information on theoretical concepts and practical applications of formulaic geometry. The research resulted in the systematization of an approach to solving geometric problems using formulas. The study revealed the peculiarities of applying the formulaic method to solve numerous classical and authorial geometric problems of varying degrees of complexity. The research obtained its main results through the use of formulaic geometry methods. The work also created new authorial construction problems, such as the composition of formulas that can solve previously unpublished problems.

$$\frac{1}{R_a} + \frac{1}{h_a} = \frac{1}{r} \qquad (1)$$

Proof: using the formula for the area of a triangle in terms of the radius of the inscribed circle and the semi-perimeter of the triangle and the formula for the inscribed circle, we get:

$$\frac{1}{R_a} + \frac{1}{h_a} = \frac{b+c}{2s} + \frac{a}{2s} = \frac{a+b+c}{2s} = \frac{P}{s} = \frac{1}{r} \quad (2)$$

Proved.

$$\frac{1}{r_a} = \frac{1}{r_b} = \frac{2}{h_c} \qquad (3)$$

Proof: we use the formula for the area of a triangle as a function of the radius of the exterior circle and the difference between the half-perimeter of the triangle and the sides, which is touched by the exterior circle:

$$\frac{1}{r} + \frac{1}{r_a} = \frac{p}{s} + \frac{p-a}{s} = \frac{2p-a}{s} = \frac{b+c}{s} = \frac{2}{R_a} \qquad (4)$$

Proved.

$$\frac{1}{r} + \frac{1}{r_a} = \frac{2}{R_a} \qquad (5)$$

Proof: we use the formulas for the area of a triangle given the radius of the exterior circle and the difference between the half-perimeter of the triangle and the sides touched by the exterior circle; we use the formula for the area of a triangle given the radius of the exterior circle and the half-perimeter of the triangle and the formula for the interior circle:

$$\frac{1}{r} + \frac{1}{r_a} = \frac{p}{s} + \frac{p-a}{s} = \frac{2p-a}{s} = \frac{b+c}{s} = \frac{2}{R_a} \Rightarrow \frac{1}{r} - \frac{1}{R_a} = \frac{1}{h_a} \qquad (6)$$

And it follows from formula (5):

$$\frac{2}{R_a} - \frac{1}{r_a} = \frac{1}{r} \qquad (7)$$

The study of the formula for the radius of an inscribed semicircle in a triangle has led to the creation of new triangle construction problems. Examples of such tasks: construct triangle ABC using the given elements.

1) R_a; p; r;
2) R_a; R; r;
3) R_a; l_a; a;
4) R_a; R_b; R_c;
5) R_a; R_b; h_c and others.

Solving complex geometric problems can positively impact active learning and students' interest in mathematics. Tasks that require the application of various theorems and properties enable students to deepen their knowledge and strengthen their understanding of mathematical principles, building confidence in their skills. Working with geometric figures can help students develop their visualization abilities and spatial imagination, which are important skills in many fields, including engineering and design. Complex tasks require a logical approach to problem-solving, which helps develop analytical thinking and the ability to make reasoned decisions. The following section will consider authorial problems in the context of a methodological approach.

Task 1

The construction of triangle ABC based on the elements h_c; h_c; l_a was proposed by Fursenko V. V. in 1937. The author suggests solving this problem using the radius of the inscribed semi-circle in the triangle (Figure 1).
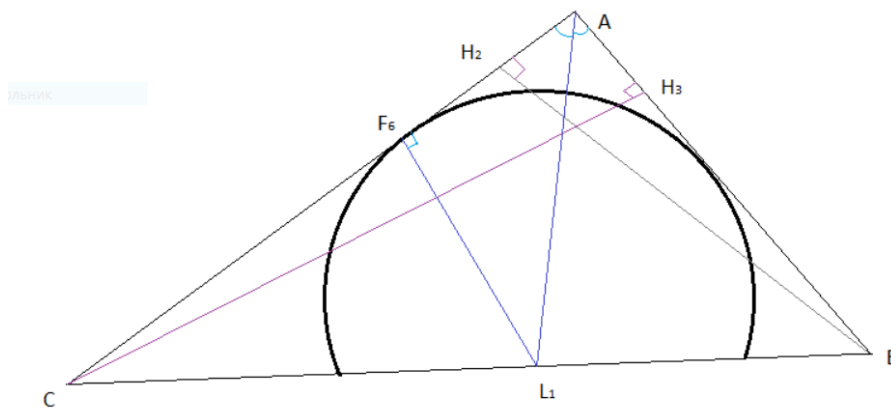

Figure 1 Graphical explanation of task 1.

Analysis:

Let triangle ABC be inscribed in a semicircle with centre L_1, belonging to side BC. Then the radius of the inscribed semicircle $R_a = L_1 F_c$ is perpendicular to side AC at the point of tangency F_c. The bisector $AL_1 = l_a$ of the internal angle $\angle BAC$ is the hypotenuse of triangle AL, F_6. Thus, triangle AL F_c can be constructed with the leg L_1, F_b and the hypotenuse AL_1. We obtain angle $L_1 A F_b$, which is equal to half of angle $\angle BAC$. Thus, the value of angle $\angle BAC$ is three times the value of angle $\angle BAC$.

Triangle AH_2B has the leg $BH_2 = h_b$ by condition and angle BAC, so we can construct and obtain side AB. Triangle AH_3C has the leg $H_3 = h_c$ by condition and angle $\angle BAC$, so we can construct and obtain side AC.

Construction:

1. Use the formula for the radius of an inscribed circle in a triangle:

$$\frac{1}{h_c} + \frac{1}{h_c} = \frac{1}{R_a} => R_a. \qquad (8)$$

2. Construct the triangle $AL_1 F_b$ by the leg and hypotenuse. We get the angle:

$$L_1AF_b = \frac{BCA}{2} => BCA. \qquad (9)$$

3. Construct the triangle AH_2B by the angle and the acute angle, and we get the side AB.
4. From the construction of triangle AH_2B by the angle and acute angle, we obtain AB.
5. From the construction of the triangle AH_3C by the angle and the acute angle, we obtain AC.
6. The desired triangle ABC can be constructed by using two sides AB and AC and BAC between them.
Constructed:

Solving such tasks can have a significant impact on active student learning and fostering their interest in mathematics for several reasons. Firstly, it involves understanding the application of mathematics. Working on geometric problems of this type shows students how mathematical principles can be applied to solve specific problems. This can help students see the value of mathematics in the real world. Secondly, such solutions are capable of developing analytical thinking. Finding a solution to a problem requires students to analyse information, engage in strategic planning, and think sequentially. These skills are fundamental to critical thinking and can be applied in many fields of knowledge and in everyday life. Thirdly, in the process of solving such problems, problem-solving skills are improved. The ability to deal with uncertainty and complexity is formed. Solving tasks similar to the one shown requires students to explore various ways to achieve their goals and choose the most effective ones.

Task 2

It was proposed by Kushnir I.A. (1991). It is recommended for solving in small groups.

A semicircle touches the sides AC and AB of triangle ABC at points F_b and F_c respectively, and its diameter belongs to the side BC. Prove that BF_b and CF_c intersect the height AH of the triangle at one point (Figure 2).
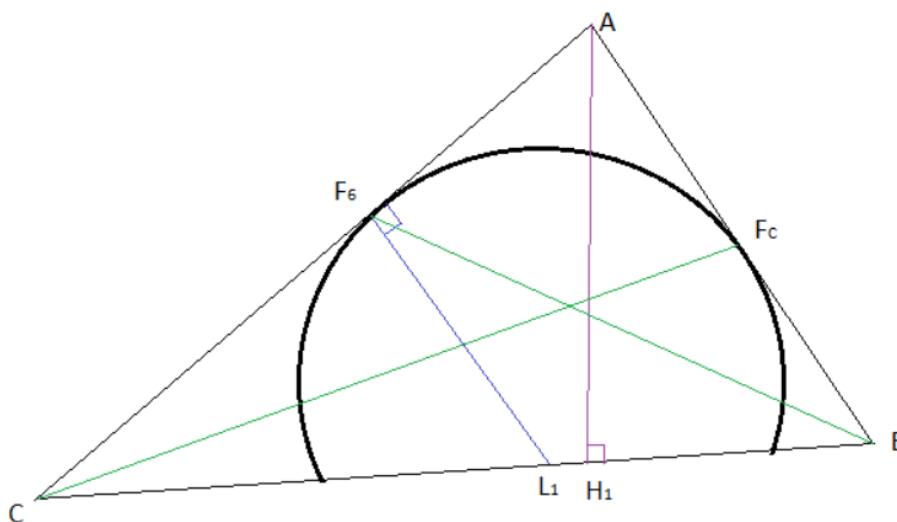

Figure 2 Graphical explanation of task 2.

Apply Ceva's theorem:

$$\frac{AF_b}{F_bC} \cdot \frac{CH_1}{H_1B} \cdot \frac{BF_c}{F_cA} = 1. \qquad (10)$$

And we will prove equality.
$AF_B=AF_C$ (as tangent segments from one point to a circle with centre L_1 and radius $L_1F_b = R_a$)=> Product (10)
It can be stated differently:

$$\frac{1}{F_bC} \cdot \frac{CH_1}{H_1B} \cdot \frac{BF_C}{1} = 1. \qquad (11)$$

From a triangle CF_bL_1=>$CF_b = R_a \cdot \mathrm{ctg} \angle C;$ \qquad (12)

from a triangle BF_cL_1=>$BF_c=R_a \cdot \mathrm{ctg} \angle B;$ \qquad (13)

from a triangle ACH_1=> $CH_1=h_a \cdot \mathrm{ctg}\angle C;$ \qquad (14)

from a triangle ABH_1=>$BH_1= h_a \cdot \mathrm{ctg} \angle B.$ \qquad (15)

Substitute the equations (12), (13), (14) and (15) into (10):

$$\frac{1}{R_a} \cdot \frac{h_a \cdot ctg \angle C}{h_a \cdot ctg \angle B} \cdot \frac{R_a \cdot ctg \angle B}{1} = 1 => 1 = 1. \qquad (16)$$

The equality is proved. So, segments BF_b and CF_c intersect with the height AH_1 of triangle ABC at one point.

Therefore, in the context of solving such tasks, the level of student engagement in learning mathematics is increased. Tasks that students find intriguing and daring can enhance their engagement in the learning process. When students actively participate in solving tasks, they are often more motivated and interested in studying the subject. Additionally, cooperation and communication are stimulated through collaborative creation of diagrams and derivation of formulas. Mathematical tasks solved in groups promote the development of communication skills and the ability to work in teams. Discussing different methods of solving and explaining one's thinking to peers can strengthen understanding of mathematics and collaboration. By solving tasks and discussing their solutions, students learn to analyse their mistakes and successes, which is an important aspect of the learning process and continuous improvement.

Task 3

Prove that the point symmetric to F_b with respect to the BC is on the same line as the points H_1 and F_c (Figure 3).
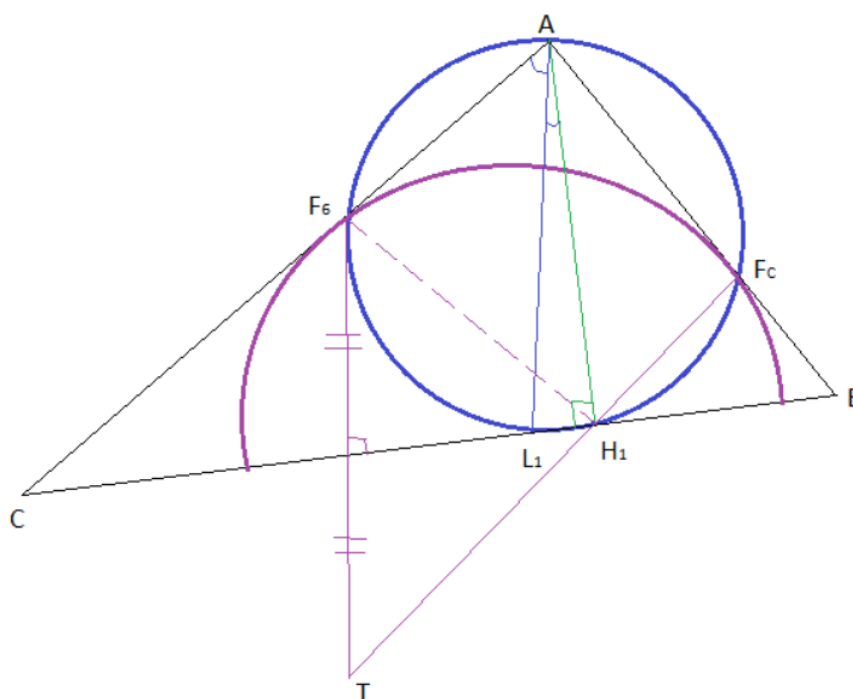

Figure 3 Graphical explanation of task 3.

Proof:
1) The point F_b is symmetrical to the point T with respect to BC => $H_1F_b = H_1T$ (because the triangle F_bH_1T is isosceles).
2) The circle circumscribed around the quadrilateral $AF_bL_1F_c$ (because $\angle AF_bL_1 + \angle AF_CL_1 = 90° + 90° = 180°$) with diameter AL, intersects the side BC at point H_1 ($\angle AH_1L_1 = 90°$). The angle $\angle AH_1L_1$ is based on the diameter AL_1.

$\angle F_bAL_1 = \angle F_CH_1L_1$ (inscribed angles resting on a single arc F_bL_1). So, the angle $\angle F_bH_1L_1 = \frac{A}{2}$.

In the triangle F_bH_1T the angle $\angle F_bH_1F_C$ is equal to the angle $\angle BAC$ ($\angle F_bH_1T = \angle BAC = \angle A$).
3) Inscribed quadrilateral $AF_bH_1F_c$ in a circle => $F_bH_1F_C = 180° - \angle A$ (by the sum of opposite angles of the inscribed quadrilateral).
4) $\angle F_CH_1F_b + \angle F_bH_1T = 180° - \angle A + \angle A = 180°$, we have points T, H_1, F_C which lie on the same line.
Proved.

The problem of finding the triangle with the minimum perimeter inscribed within another triangle has always intrigued humanity. One such triangle is the orthocentric triangle – a triangle whose vertices lie on the base of a given triangle. For example, the triangle $H_1H_2H_3$ is orthocentric to triangle ABC (Figure 4).
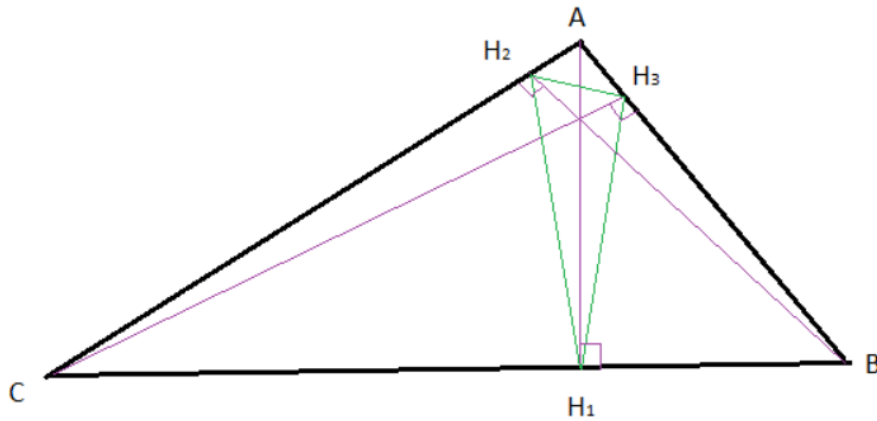
Figure 4 Explanation of task 3

.

Such tasks allow students to apply theoretical knowledge in practice, which helps to strengthen their understanding of mathematical concepts. Solving geometry problems often requires logical reasoning and critical analysis, which improves these skills.

Task 4

Prove that among the triangles inscribed in a semicircle with a diameter belonging to side BC and touching the other two sides AC and AB, triangle $F_b H_1 F_C$ has the smallest perimeter (Figure 5).
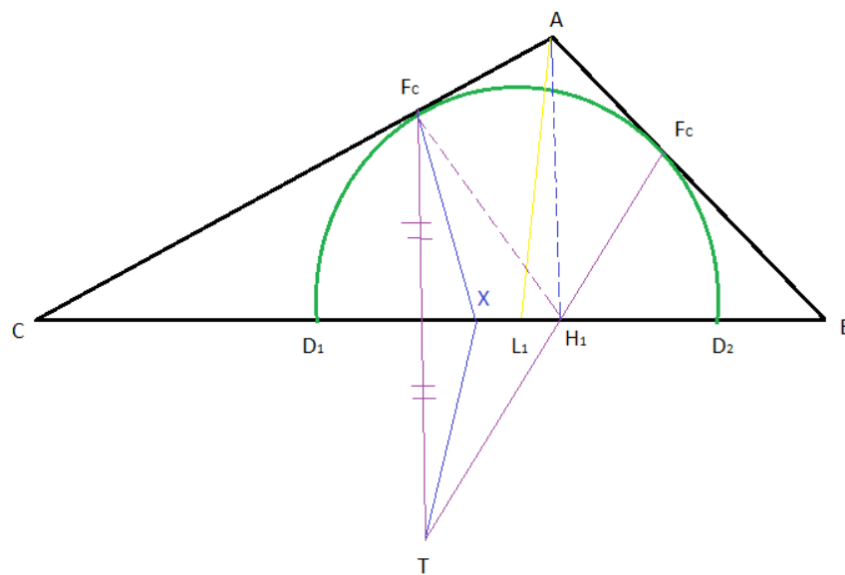

Figure 5 Explanation of task 4.

Proof:

Based on Task 3, we have an isosceles triangle $F_b H_1 T$. Let X be any point of diameter $D_1 D_2$ of the semicircle inscribed in triangle ABC (not coincident with point H_1). Denote the perimeter of triangles $F_b H_1 F_c$ and $F_b X F_c$ by P and P_1, respectively. Then,

$$P = F_b H_1 + H_1 F_C + F_b F_C = T H_1 + H_1 F_C + F_b F_C; \quad (17)$$

$$P_1 = F_b X + X F_C + F_b F_C = T X + X F_C + F_b F_C \quad (18)$$

Compare (17), (18), since the length of the broken TXF_C is always longer than the length of the straight TF_C, where H_1 is TF_C, then P<P_1.

Otherwise, the triangle $F_C H_1 F_C$ inscribed in a semicircle has the smallest perimeter.

Proved.

Searching for the solution to such a task develops students' ability to break down large and complex problems into smaller, manageable parts. Geometric tasks help students better visualize and understand abstract concepts. Working on such tasks can be more engaging and motivating than passive theory study, as students see the immediate results of their efforts.

Successfully solving a task can bolster students' confidence in their abilities, which, in turn, can increase their interest in the subject.

Task 5

Triangle ABC contains three semicircles with centres $L_1 L_2 L_3$, touching its sides at points $F_b, F_C; Q_a, Q_C; T_a, T_b$.
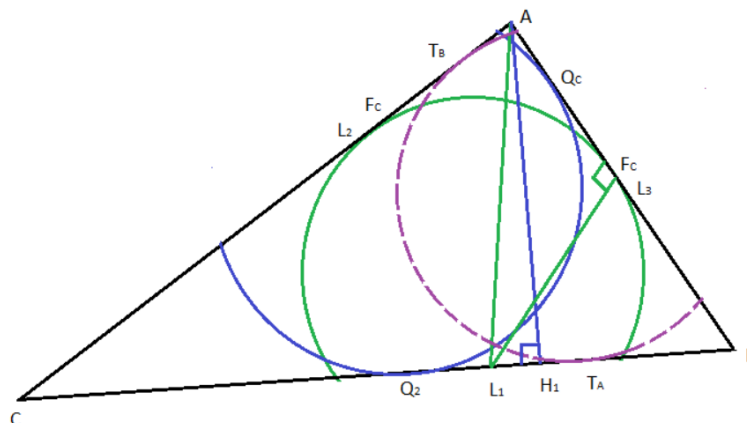

Figure 6 Explanation of task 5.

Prove that:

$$\frac{T_b C}{F_b B} \cdot \frac{Q_c A}{Q_a C} \cdot \frac{T_a B}{T_a A} = 1 \qquad (19)$$

Proof:
From the similarity of the triangles $L_1 F_c B$ and $A H_1 B$ ($\angle L_1 F_c B = \angle A H_1 B = 90°, \angle L_1 B H_1 = \angle A H_1 B = \angle B$) we have:

$$\frac{F_c B}{H_1 B} = \frac{R_a}{h_a} \qquad (20)$$

and

$$\frac{F_b C}{H_1 C} = \frac{R_a}{h_a}. \qquad (21)$$

From:

$$\frac{F_c B}{H_1 B} = \frac{F_b C}{H_1 C} \qquad (22)$$

or

$$\frac{F_c B}{F_b C} = \frac{H_1 B}{H_1 C}. \qquad (23)$$

Similarly:

$$\frac{Q_a C}{Q_a C} = \frac{H_2 C}{H_2 A}, \qquad (24)$$

$$\frac{T_b A}{T_a B} = \frac{H_3 A}{H_3 B}. \qquad (25)$$

Multiply (23), (24), (25):

$$\frac{F_c B}{F_b C} \cdot \frac{Q_a C}{Q_c A} \cdot \frac{T_a A}{T_a B} = \frac{H_1 C}{H_1 C} \cdot \frac{H_2 C}{H_2 A} \cdot \frac{H_3 A}{H_3 B}. \qquad (26)$$

From the triangle $L_1 F_c$ B :

$$F_c B = R_a \, \mathrm{ctg} \angle B. \qquad (27)$$

Similarly:

$$F_b C = R_a \, \mathrm{ctg} \angle C; \qquad (28)$$

$$Q_a C = R_b \cdot \mathrm{ctg} \angle C ; \qquad (29)$$

$$Q_c A = R_b \cdot \mathrm{ct\, g} \cdot \angle B ; \qquad (30)$$

$$T_b A = R_c \cdot \mathrm{ctg} \angle A ; \qquad (31)$$

$$T_a B = R_c \cdot \text{ctg} \angle B. \qquad\qquad (32)$$

Substitute into the product (26):

$\frac{R_a \cdot \text{ctg} \angle B}{R_a \cdot \text{ctg} \angle C} \cdot \frac{R_b \cdot \text{ctg} \angle C}{R_b \cdot \text{ctg} \angle A} \cdot \frac{R_C \cdot \text{ctg} \angle A}{R_C \cdot \text{ctg} \angle B} = 1$, because $\frac{H_1 B}{H_1 C} \cdot \frac{H_2 C}{H_2 A} \cdot \frac{H_3 A}{H_3 B} = 1$ (by Ceva's theorem).

We get (32) 1=1. Proved.

Solving complex tasks can encourage the exchange of ideas and collaboration among students, promoting deep learning and the development of social skills. It also teaches students important life skills such as perseverance and resilience.

By incorporating these principles into mathematics teaching, teachers can significantly enhance students' interest, motivation, and active participation in the learning process. Active learning is essential in delivering mathematics education as it encourages students to engage in the learning process, enhances their understanding of mathematical concepts, develops their communication skills, and stimulates their interest in the subject.

During discussions, there is an ongoing debate about various methods of active learning in mathematics education, such as group projects, discussions, and problem-based tasks, and their effectiveness and impact on understanding and assimilating mathematical concepts. Discussions may also arise regarding the role of active learning in solving mathematical problems using original solution methods based on new dependencies previously unknown. This can promote the development of teaching practices and enhance the quality of students' logical thinking (Frank et al., 2021).

5. Conclusions

Active learning is essential for students to effectively understand mathematical concepts. Stimulating students' interest in mathematics is vital for their successful learning. Using practical examples and supporting independent problem-solving can help engage students and demonstrate the practical applicability of mathematical concepts. Formative solving of geometric problems significantly reduces the size of proofs. Using formulas to calculate the radius of the inscribed semicircle in a triangle can help to focus on the main ideas of the problem and perform calculations more efficiently. This precise approach to problem-solving can help to avoid unnecessary repetition of calculations and details, which can be time-consuming and can significantly reduce the volume of the solution.

Further research and practical initiatives are necessary to improve the quality of mathematical education and support the successful development of students. This includes the development of new methods and strategies that meet evolving learning needs and support the progressive development of the educational system. Further research and development of new techniques and formulas aims to enhance the quality of mathematical education and provide the skills necessary for successful learning and professional development. Relevant material can be the subject of research and creative work in geometry.

Ethical considerations

Not applicable.

Conflict of Interest

The authors declare no conflicts of interest.

Funding

This research did not receive any financial support.

References

Burns, M., & Ostfeld, D. (2020). *Active learning in mathematics: Creating engaging learning environments.* Columbia University Press.

Burns, M., & Ostfeld, D. (2020). Active Learning Strategies in Mathematics Education. *Journal of Mathematics Education*, 45(2), 112-125.

Davis, V., Reddy, D., & Lowy, K. (2020). Active Learning Pedagogies in Higher Mathematics Education. *Journal of Mathematical Pedagogy*, 20(1), 45-58.

Dubrovin, B.A., Novikov, B.A., Fomenko, A.T. (1986). *Modern geometry: methods and applications*. Nauka, 320 p.

Fomin, S.V., Shafarevych, Y.R. (1986). *Problems of higher algebra and geometry*. Nauka, 368 p.

Frank, M., Maxwell, D., & Carbon, G. (2021). *Active learning in mathematics: Applications and methods.* Routledge.

Frank, M., Maxwell, D., & Carbon, G. (2021). Implementing Active Learning Techniques in the Mathematics Classroom. *Mathematics Teaching Strategies Quarterly*, 30(3), 175-189.

Hartshorne, R. (1999). *Algebraic geometry*, 496 p.

Herndon, L., & Weiss, S. (2021). Creating Interactive Mathematics Classrooms: Strategies for Engaging Students. *Mathematics Education Research Journal*, 33(2), 87-102.

Holubiev, V.V., Kondratenko, V.S. (1983). *Elementary Geometry*. Kyiv: Higher School, 272 p.

Iatsyshyn, A., Iatsyshyn, A., Kovach, V., Radchenko, O., Turevych, A. (2020). Application of open and specialized geoinformation systems for computer modelling studying by students and PhD students. *CEUR Workshop Proceedings*, 2732, 893-908.

Johnson, J. (2020). Enhancing Student Learning Through Active Learning Methods in Mathematics. *International Journal of Mathematical Education in Science and Technology*, 51(3), 345-358.

Khimich, O.P., Chornobai, V.V. (2017). *Collection of problems in mathematics*. Kyiv: Genesis, 224 p.

Kostriuk, A.H., Kramer, N.A. (1981).*Vectors and Tensors in Physical and Geometric Space*. Kyiv: Naukova Dumka, 304 p.

Kostrykin, A.I., Manyn, Yu.Y. (1986). *Linear Algebra and Geometry*. Nauka, 432 p.

Kovalchuk, T.M., Maksymiuk, O.H. (2017). *Collection of problems in geometry for grades 10-11*. Kyiv: Osvita, 240 p.

Kushnir, I.A. (1991). *Triangle and tetrahedron in problems*. Kyiv, 208 p.

Kushnir, I.A., Kushnir, T. (1991). *Triangle and tetrahedron in tasks*. Kyiv: Soviet School, 208 p.

Levin, J., & Wilson, J. (2022). Student Engagement and Achievement in Mathematics: The Role of Active Learning. *Journal of Educational Psychology*, 115(2), 89-104.

Ly, P. (1980). *Geometry and Lie groups*, 448 p.

Mamontov, Yu.I., Sydorenko, S.I. (1988). *Theory and Problems in Algebra and Beginnings of Analysis*. Kyiv: Higher School, 352 p.

Mamontov, Yu.I., Sydorenko, S.I. (1989). *Theory and Problems in Geometry*. Kyiv: Vysha Shkola, 352 p.

Martin, D., & Griffin, S. (2022). Active Learning Techniques for Developing Critical Thinking Skills in Mathematics. *Journal of Critical Mathematics Education*, 15(2), 112-127.

Milnor, J. (1984). *Topology from a geometric point of view*, 332 p.

Morris, D., & Fitzgerald, S. (2020). Active Learning Strategies for Fostering Mathematical Thinking in Students. *Mathematics Education Review*, 25(3), 211-225.

Peterson, I.N. (1999). *Geometry. Textbook for 10th grade*. Kyiv: Pedagogichna Dumka, 272 p.

Petrov, V.K. (2016). *Geometry. Problems with solutions for grades 10-11.* Kyiv: Genesis, 256 p.

Popov, O. O., Kyrylenko, Y. O., Kameneva, I. P., Iatsyshyn, A. V., Iatsyshyn, A. V., Kovach, V. O., Kiv, A. E. (2022). The use of specialized software for liquid radioactive material spills simulation to teach students and postgraduate students. *Paper presented at the CEUR Workshop Proceedings*, 3085, 306-322.

Robinson, D. A., & Elinbee, D. M. (2022). The Impact of Active Learning on Student Engagement and Achievement in College Mathematics. *Journal of Higher Education Mathematics*, 38(4), 289-304.

Semenova, T.H., Shvab, A.M. (2018). *Geometry. 10th grade: Textbook for secondary schools*. Kyiv: Osvita, 320 p.

Serhiienko, I.V. (1988). *Functional analysis: Linear Operators in Hilbert Spaces*. Kyiv: Vysha Shkola, 336 p.

Skiba, R.V. (2018). *Geometry. Textbook for the 10th grade of secondary schools*. Kyiv: Genesis, 352 p.

Sokolov, O.V., Yashchenko, M.S. (2018). *Geometry. Collection of problems for*. Kyiv: Osvita, 192 p.

Tracy, D., & Shellenberg, S. (2021). Active Learning Approaches to Improve Student Success in Mathematics. *Journal of College Mathematics Instruction*, 28(4), 245-259.