# EDUCATIONAL PROCESS

INTERNATIONAL JOURNAL

ISSN 2147-0901 e-ISSN 2564-8020

Volume 16 (2025)

ÜNİVERSİTEPARK®

**Educational Process: International Journal**

An official publication of the Universitepark Limited

Publication Office: Universitepark Ltd, iTOWER Plaza (№61), Merkez Mh Akar Cd №3, Sisli, Istanbul, Türkiye, 34382

Publisher WebSite: www.universitepark.com.tr

Publisher Email: info@universitepark.com.tr

Journal Web: https://edupij.com/

Journal Email: editor@edupij.com



**ISSN** 2147-0901 (Print), 2564-8020 (Online)

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ISSN 2147-0901 (Print), 2564-8020 (Online)

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ISSN 2147-0901 (Print), 2564-8020 (Online)

EDUCATIONAL PROCESS INTERNATIONAL JOURNAL

## Research Article

**Cite this article:** Hetmanenko, L. (2025). Exploring Additivity in Triangle Areas: Methodological Insights for Mathematics Education. *Educational Process: International Journal, 16,* e2025247. https://doi.org/10.22521/edupij.2025.16.247

**Received** April 7, 2025
**Accepted** May 8, 2025
**Published Online** June 13, 2025

**Keywords:** Educational process, additivity of areas, analogy, centroid, orthocenter, incentre, euler's line, visualization technologies, digital platforms.

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# Exploring Additivity in Triangle Areas: Methodological Insights for Mathematics Education

**Liudmyla Hetmanenko**

## Abstract

**Background/purpose.** In modern mathematical education, it is important to develop students' ability to understand the fundamental properties of geometric objects deeply. This makes it relevant to study the additivity of the area of triangles as a property inherent in various kinds of quantities and ways of representing it methodologically in the process of teaching mathematics. This article aims to define the theoretical basis and form methodological approaches to the study of the additivity of the area of triangles.

**Materials/methods.** In the course of the study, literature sources were synthesized to form the theoretical foundations of the additivity of triangle areas, a comparative analysis was carried out to compare different digital platforms for visualizing educational materials in mathematics education, and a pedagogical experiment was conducted to determine the effectiveness of the methodological approach in teaching the additivity of triangle areas to students of mathematical specialities.

**Results.** In the control group, the average value of the final grades was Mean= 8.091, while in the experimental group the average score was Mean = 9.455; therefore, students who studied according to the improved methodology have a relatively higher level compared to traditional methods, which confirms the expediency of using digital tools in the educational process, which contributes to better learning and reduces the variance of student performance.

**Conclusion.** The use of interactive methods contributes to a more systematic mastery of the concept of additivity in the area of triangles, which has further application in the study of the principles of geometric additivity and related topics.

## 1. Introduction

In modern mathematical education, it is important to develop students' ability to deeply understand the fundamental properties of geometric objects, including their quantitative characteristics. One of these key properties is the additivity of the area of triangles, which plays a crucial role in solving problems related to geometric modelling, analytical geometry and measurement theory.

Given the growing use of digital tools in mathematics education, there is a need to improve methodological approaches to teaching, including the topic of additivity of the area of triangles, which will contribute to the development of students' analytical and spatial thinking. This makes it important to study the additivity of the area of triangles as a property inherent in various kinds of quantities and ways of its methodological representation in the process of teaching mathematics.

Digital visualization technologies in mathematics education are currently being emphasized as they can create interactive models and visual representations of complex mathematical concepts. In addition, such tools significantly improve students' understanding of abstract ideas such as geometric shapes, which is especially important for learning topics such as the area of triangles and their additive properties (Bilousova et al., 2022). Digital technologies are a relatively common method of engaging students actively in learning initiatives. According to a survey conducted by Statista in 2023, the main reasons for using digital technologies in education are to increase efficiency, motivate students and improve learning outcomes (Korhonen, 2024) (Figure 1).

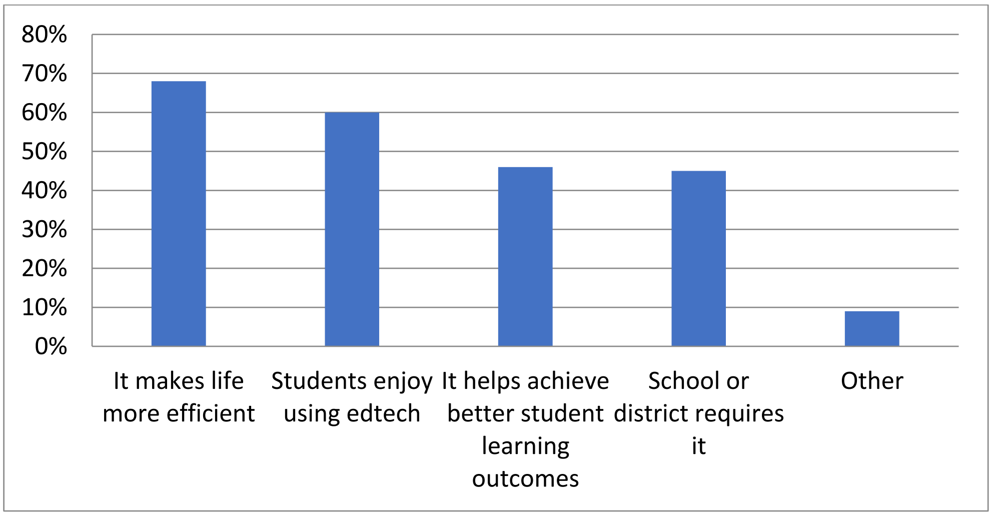


**Figure 1.** Advantages of Using Digital Technologies in Education

Source: Korhonen (2024).

The results of a survey conducted as part of the study by Zhylin et al. (2023) show that currently, the majority of teachers (approximately 79%) use digital tools regularly, which has significantly changed the interaction between teachers and students to a comprehensive horizontal communication of participants in the educational process. However, Bakhmat et al. (2023) note that the degree of development of digital technologies in higher education institutions leads to a significant gap in the level of professionalism in general and digital competence, which requires an integrated approach to developing educational programmes based on equality. Thus, given the popularity of digital technologies among teachers, which is confirmed by high rates in previous studies, it is important to improve methodological approaches to studying fundamental mathematical concepts through visualization and interactive tools.

This research article aims to define the theoretical basis and form methodological approaches to the study of the additivity of the area of triangles. It also seeks to improve the methodology and

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methods of presenting materials in mathematical education. The aim of this article is to investigate the effectiveness of the use of digital technologies in teaching the additivity of the area of triangles to students of mathematical specialities. The article also includes a description of the process of implementing the proposed methodological approach to the study of the additivity of triangles and determining its effectiveness in teaching the additivity of the area of triangles to students of mathematical specialities. Therefore, this study aims to answer the following questions:

- What conceptual approaches to the principle of additivity of the area of a triangle are distinguished in the modern scientific literature?
- Do interactive methods (using GeoGebra and Desmos Geometry) ensure the quality of learning and systematic learning of the additivity of the area of a triangle?
- What are the prospects for integrating digital tools into mathematics teaching to achieve a long-term effect?

## 2. Literature Review

In mathematical science, the general property of quantities in algebra and geometry is still relevant. For example, Kaufmann (2014) note that the study of triangle areas can be related to bilinear forms on points in the complex plane, whereas individual triangle areas can be derived from sets of points on algebraic curves. Instead, Damásdi et al. (2019) investigate the additive properties of triangles in combinatorial geometry, particularly, the distribution of their areas in in-plane line arrangements, using the results of additive combinatorics. However, Zhang et al. (2018) point out that additivity is a fundamental property of mathematical objects that determines their quantitative relationships and structural characteristics, particularly in triangle areas, where it manifests itself through specific properties. In the majority of cases, such properties are confined to the specific field of study. For instance, in Aziz et al. (2023), Jordan maps on triangular rings that force additivity were generalized. Furthermore, the study of additivity of maps in triangular rings in terms of the process of supporting additive properties by mathematical structures that may be parallel to the additivity of triangle areas was advanced by Aziz et al. (2024). Consequently, additivity as a property inherent in various quantities is particularly interesting.

The additivity of the arcs of the Euler circle is known and studied earlier. As noted by Hetmanenko (2023), the additivity of arcs occurs in any circle described around a triangle. For example, Pamfilos (2020) analyzed the properties of triangles with a given described circle and Euler circle, which are a one-parameter family, to determine triangles of maximum area/perimeter. Croft et al. (2012) noted that the additivity of arcs in circles described around triangles is directly related to the preservation of angular relations and the invariance of arc lengths under affine transformations, which allows generalizing such properties to broader classes of geometric configurations. Since additivity is inherent in various kinds of quantities, it is an important tool for studying the relationships between geometric and algebraic properties, particularly in the context of Euler circle arcs (Kaufmann, 2014). Chow et al. (2020) describe unique solutions, such as the contraction circle, which are related to geometric quantities such as curvature, length, area, and support function and, in the context of triangles, can take into account the additive properties of their respective characteristics. Consequently, these properties allow you to create more versatile models for analyzing a variety of geometric situations, including optimizing the areas and perimeters of triangles in different spaces.

In terms of ensuring the quality of teaching the additivity of the area of triangles in mathematics education, an important aspect is the introduction of interactive teaching methods that allow students to learn theoretical foundations by applying them in practice (Caviedes et al., 2023; Machado et al., 2023; Polotskaia & Savard, 2021; Yeung & Ng, 2023). New technologies for visualizing geometric shapes significantly improve students' perception of mathematical quantities to develop new knowledge and replace old methods; for example, Herbst (2003) suggests using a task to compare cardboard triangles to develop a mathematical understanding of the area. Instead, modern authors propose the use of dynamic geometric programmes such as GeoGebra to study the area of triangles through variable parameters (Bilousova et al., 2022; Juandi et al., 2021) and the modelling of spatial figures in Desmos Geometry, including the analysis of their additive properties (Aksu & Zengin, 2022; Machado et al., 2023). In contrast, Machromah et al. (2019) tested the benefits of GeoGebra for university students studying and practising calculus and found that it provides a meaningful discourse. In another study, Haciomeroglu et al. (2009) and Ziatdinov & Valles Jr (2022) showed that GeoGebra helped teachers become familiar with the digital use of geometry, algebra and calculus concepts that are commonly taught to university students. The use of modern technologies, in particular computer modelling and visualization of geometric shapes, can significantly improve the understanding of the additive properties of triangles, as confirmed by several studies in the field of mathematics didactics (Aksu & Zengin, 2022; Bilousova et al., 2022; Yeung & Ng, 2023; Žakelj & Klancar, 2022). In addition, it is also important to provide a systematic approach to learning that includes steps from learning the theoretical foundations of triangle area additivity to exploring more complex additive properties, in particular through the context of combinatorial geometry, as outlined by Ye et al. (2022), which, according to Koch et al. (2019), allows for a fair comparison of a wide range of geometric learning algorithms.

## 3. Methodology

### *3.1. Data Collection*

Data was collected from both theoretical and empirical sources.

The theoretical basis was established through a synthesis of 68 peer-reviewed articles indexed in Scopus and Web of Science, published between 2005 and 2024. The selection criteria included relevance to mathematics education, geometry didactics, and the use of digital tools in pedagogy. Keywords such as "additivity of area," "triangle geometry," "digital learning platforms," and "mathematics visualization" were used during database searches.

Empirical data were obtained through a pedagogical experiment involving 66 students of mathematical specialities. The participants were randomly divided into a control group (n = 33) and an experimental group (n = 33). Both groups took part in two teaching sessions. The control group received traditional instructional materials, while the experimental group was taught using a revised digital-based curriculum. The primary data consisted of students' final assessment scores after the intervention.

### *3.2. Data Preparation*

Theoretical data were systematized by categorizing the selected literature according to thematic relevance: theoretical background, methodological approaches, empirical validation, and digital platform application in education. Bibliographic information was structured using reference management software (Zotero).

Empirical data from the pedagogical experiment was anonymized and entered into a spreadsheet. Inconsistencies such as incomplete test scores or duplicated entries were removed. All data were formatted according to the requirements of the JASP statistical software, ensuring compatibility for further analysis. Variables were standardized to include group designation (control/experimental), score outcomes, and session metadata.

### *3.3. Data Analysis*

Descriptive statistical methods were applied using JASP software to analyze the academic performance of both groups. Measures of central tendency (mean, median) and variability (standard

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deviation) were calculated to determine differences in learning outcomes. Comparative analysis was used to assess the effectiveness of the digital platform in enhancing students' understanding of the additivity of triangle areas. Findings were interpreted in the context of both the theoretical framework and pedagogical objectives, allowing for evidence-based conclusions regarding the impact of digital tools in mathematics education.

## 4. Results

### *4.1. Theoretical Foundations of Additivity of Areas of Triangles*

The additivity of the areas of triangles is mainly based on the finite additivity property. Given the finite additivity property, it should be noted that the area of an object divided into a finite number of non-intersecting parts is equal to the sum of the areas of these parts. If we consider additivity as a fundamental property, it is important to formalize the statement that a geometric figure can be divided into a finite number of non-intersecting regions with defined boundaries (Bevz, 2005; Bingham, 2010). Therefore, any triangle can be divided into a finite number of non-intersecting triangles for which the additivity property guarantees the preservation of the total area. In other words, drawing an auxiliary line makes it possible to represent the area of the original triangle as the sum of the areas of the resulting triangles, which follows from the additivity of area and the properties of triangle division (Kushnir, 2007). In this context, the axiom of area additivity allows us to justify the equality of the areas of a figure's parts when divided into non-intersecting regions, which is an important method of proving geometric theorems. That is, if a triangle T is divided into two triangles, T1 and T2, by a line segment connecting a vertex to a point on the opposite side, then the area of T is equal to the sum of the areas of T1 and T2 (May 2001). As a result, the property is widely used to calculate the area of triangles using various methods and to solve problems involving the expression of area through other parameters of a triangle.

### *4.2. Methodological Approaches to Studying the Additivity of the Area of Triangles*

The need to develop methodological approaches to the study of the additivity of the area of triangles, which is one of the fundamental properties for dividing complex geometric figures into elementary parts, is due to the importance of systematically updating higher education curricula to ensure the quality of mathematical education. The following applied aspects, which include the study of the general case and the proof of the additivity of the area of triangles, in particular, the analysis of individual cases using the classical centres of a triangle and their generalization to the general case, will allow systematizing the methods of teaching and learning by students the process of proving and solving problems of this category and will contribute to a deeper understanding of the relationships between geometric quantities in triangles in mathematical education. Below, we offer our own interpretation of the general case, which is supported by a number of practical tasks proposed for students of mathematical specialities of the first (bachelor's) level to prove the additivity of the area of triangles.

**General case.** The point$\mathrm{M}$ of the triangle $\Delta\mathrm{ABC}$ , which is the point of intersection of the medians, is called the centroid, which divides each median in the ratio $2:1$ , with the longer segment closer to the vertex of the triangle.

Draw an arbitrary line through the point $\mathrm{M}$ , where the distances from the vertices of the triangle to this line are additive:

$$\rho_1 + \rho_2 = \rho_3 \tag{1}$$

The equation above expresses the condition that the sum of two specific distances from the vertices of the triangle $\Delta\mathrm{ABC}$ to the corresponding line is equal to the third distance. Thus, it reflects

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the relationship between spatial distances in a triangle, where each distance is defined as a perpendicular from a vertex to the opposite side.

Consider the case when this line intersects the sides $AB$ and $AC$.

Let us prove that the sum of the perpendicular distances from the vertices $B$ and $C$ to the line $BC$ equals the perpendicular distance from the vertex $A$ to the same line. This geometric property ensures that perpendicular distances in a triangle are constant:

$$\rho_B + \rho_C = \rho_A \tag{2}$$

where $\rho_B$ is the distance from the vertex $B$ of the triangle $ABC$ to this line;

$\rho_C$ is the distance from the vertex $C$ to this line;

$\rho_A$ is the distance from the vertex $A$ of the triangle $ABC$ to this line.

A visual representation of the $ABC$ triangle is shown in Figure 2.

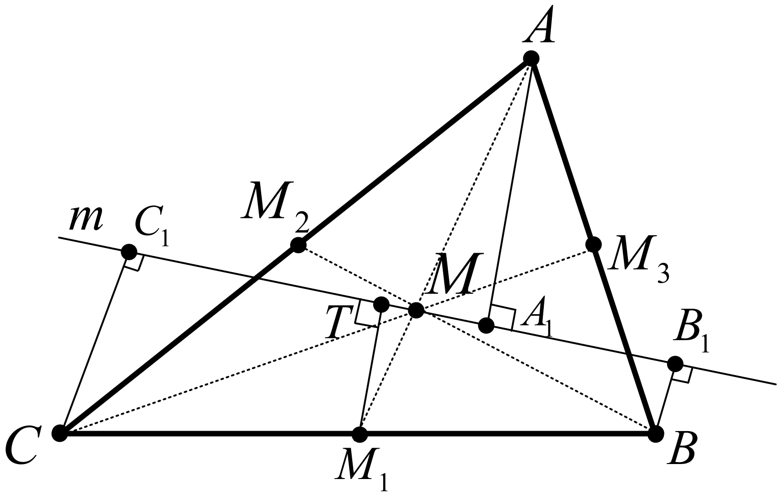


**Figure 2.** Visual Representation of a Triangle $ABC$

You are given a triangle $\Delta ABC$ , where the point $M$ is the centroid and denotes the intersection of its medians. It is known that any median is divided by this point with respect to $2:1$ , where most of the segment is adjacent to the vertex of the triangle. Also, the linem generally passes through the centroid point ($M \in m$ ), and $CM_1$ is the median of the triangle ($CM_1 = M_1B$ ). Therefore, let us denote the point of intersection of the line with the side $AC$ as $M_1$ , i.e.$M_1 = m \cap AC$ and $T = AB \cap m$, respectively.

We also note the orthogonal projections of the triangle's vertices on the line$m$ . Given that the points , ,$A_1B_1C_1$ are the corresponding projections of the vertices $AB$ and $C$ on the line $mCC_1 \perp mBB_1 \perp m$ , and$AA_1 \perp m$ . The point $T$ is the midpoint of the segment $M_1M_3$, where$M_3$ is the corresponding constructed auxiliary point. The midpoint of the rectangular trapezoid formed by the line$m$ is $M_1T$ , i.e. .$M_1T \perp m$.

So, we need to prove that the distances from the triangle's vertices to this line are additive. We assume that they are under the following condition:

$$\rho_A = \rho_B + \rho_C \tag{3}$$

In this case, $CC_1 = \rho_C BB_1 = p_B$ and $AA_1 = \rho_A$

**Proof**. The perpendiculars $CC_1$ and $BB_1$ to the line$m$, drawn from the vertices $C$ and $B$ of the triangle $ABC$, form a rectangular trapezoid $CC_1B_1B$ . The bases of this trapezoid are $\rho_C$ and $p_B$ respectively. Then $M_1T$ is the midline of the trapezoid $CC_1B_1B$, where $CM_1 = M_1B$ and $M_1T \perp C_1B_1$ are the vertices (Figure 2). This can be represented by the equation:

$$M_1T = \frac{\rho_B + \rho_C}{2}$$

To do this, we rely on the property of medians in a triangle, namely:

$$AM: MM_1 = 2: 1$$

Hence, taking into account the properties of the median, we obtain:

$$AA: M_1T = 2: 1$$

$$AA_1 = 2M_1T$$

$$AA_1 = p_B + p_C = p_A$$

From $AA_1 = \rho_A$ , the following variant of the equality is possible:

$$CC_1 + BB_1 = p_B + p_C = p_A$$

It is proved that the sum of the distances from two vertices of a triangle to a line passing through its centroid is equal to the distance from the third vertex of the triangle to the same line. This additivity of distances from the vertices of a triangle to a line passing through the point of intersection of the medians makes it possible to create triangles with a common side lying on such a line.

Thus, the areas of such triangles will be additive - this is **a general case of additivity of triangle areas,** as shown in Figure 3.

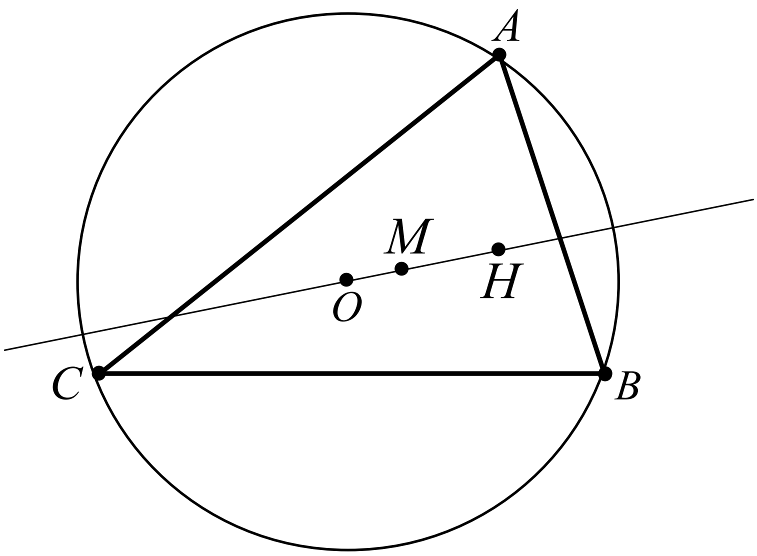


**Figure 3.** General Case of Additivity of Areas of Triangles

**Example 1:** The point $M$ is the centroid of the triangle $ABC$ . Let us draw the Euler line, which is the centre of the described circle $O$ , with the orthocentre at the intersection of the heights $H$, the centroid $M$ at the intersection of the medians. A visual representation of the triangle is shown in Figure 3.

Consider the line $OH$ in Figure 4, which intersects the sides $AC$ and $AB$ of triangle $ABC$ . Then you need to prove that the triangles $\Delta AOH$ $\Delta BOH$ and $\Delta COH$ areas are additive. That is, prove that the following equality holds:

$$S_{AOH} = S_{BOH} + S_{COH} \tag{4}$$

This equality confirms the additivity of the areas of the defined triangles based on the properties of the Euler line.

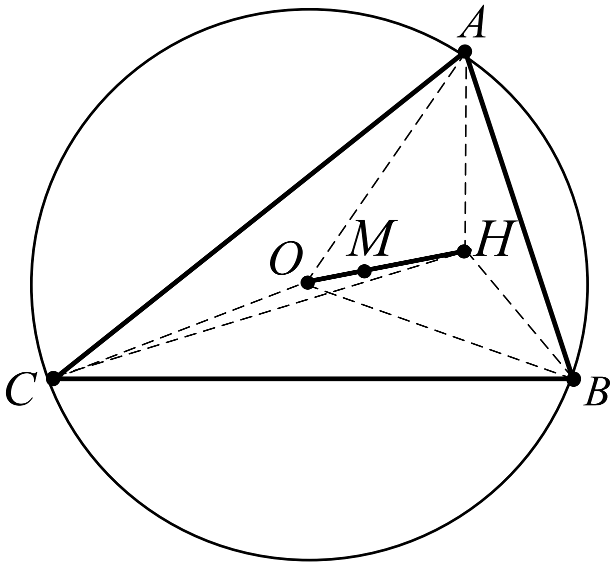


**Figure 4.** The Straight Line OH

**Proof**. It is known that the Euler line contains the orthocentre at point $H$ , the centre of the circumscribed circle at point $O$ and the centroid at point $M$, which follows from Euler's theorem. The centroid divides the medians with respect to $2:1$ , and the Euler line passes through it, which provides its unique properties in the context of area distribution.

The line $OH$ , the Euler line, passes through the point $M$ of the triangle $ABC$. Since $OH$ is the common side of triangles $\Delta AOH, \Delta BOH$, and $\Delta COH$, the areas of these triangles can be expressed as the heights to the common side $OH$. The distances from the vertices $AB$ and $C$ to the line $OH$ are additive, since:

$$AA_1 = \rho_A = \rho_B + \rho_C$$

In this case, we will use the classical triangle area formula, which allows us to express the areas of triangles in terms of their heights, which is key to proving the additivity of areas.

For the triangle $\Delta AOH$ , the formula looks like this:

$$S_{AOH} = \frac{1}{2} OH * AA_1 = \frac{1}{2} OH * \rho_A$$

For the triangle $\Delta BOH$, a similar formula is used:

$$S_{BOH} = \frac{1}{2} OH * BB_1 = \frac{1}{2} OH * \rho_B$$

For the triangle $\Delta COH$ , respectively:

$$S_{COH} = \frac{1}{2} OH * CC_1 = \frac{1}{2} OH * \rho_C$$

where $OH$ is the length of the base;

$\rho_A, \rho_B, \rho_C$ are the height of the triangle drawn from the vertices $ABC$ to the line $OH$.

A visual representation of the expression of the area of triangles through their heights is shown in Figure 5.

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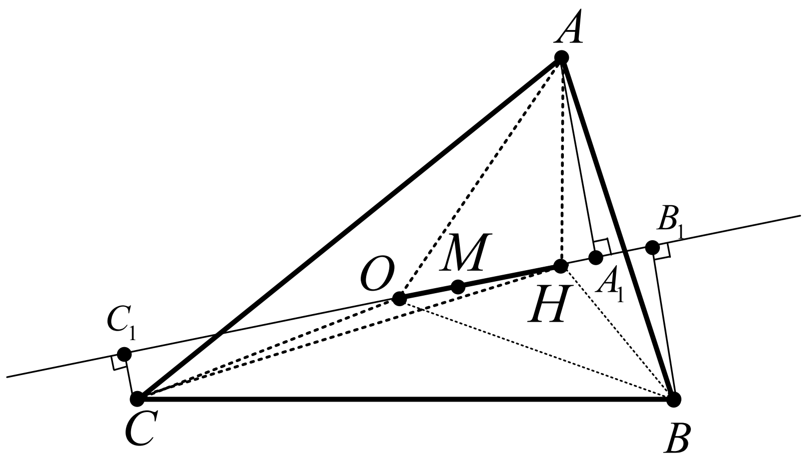


**Figure 5.** Visual Representation of the Expression of the Area of Triangles Through Their Heights

Given that the sum of the distances $\rho_B$ and $\rho_c$ is equal to $\rho_A$ , which follows from of the geometric properties of the points under consideration, adding the equations for the areas of the triangles $\Delta BOH$ and $\Delta COH$ , we obtain the following equality:

$$S_{BOH} + S_{COH} = \frac{1}{2}OH(BB_1 + CC_1) = \frac{1}{2}OH(\rho_B + \rho_C)$$

Thus, substituting the value of $\rho_A$ into the data, we get the equation:

$$\frac{1}{2}OH(\rho_B + \rho_C) = \frac{1}{2}OH * \rho_A = S_{AOH}$$

The equation proves that the area of triangle $\Delta AOH$ is equal to the sum of the areas of triangles $\Delta BOH$ and $\Delta COH$. In general, it can be represented as follows:

$$S_{BOH} + S_{COH} = S_{AOH}$$

Thus, the equality of the areas of triangles $\Delta AOH$ $\Delta BOH$ and $\Delta COH$ , which follows from the properties of the Euler line and the additivity of triangle areas, demonstrates the location of the key centres of the triangle on the same line, as well as the fundamental relationship between the geometric parameters of the triangle.

**Example 2:** The Euler line $OH$ (where $OM: MH = 1: 2$ ) contains the centre of the Euler circle, the circle of nine points, the circle passing through the bases of the heights of the triangle, the midpoints of the sides of the triangle and the midpoints of the segments $AHBHCH$ (where $AB$ and $C$ are the vertices of the triangle $\Delta ABCH$ is the orthocentre of the triangle $ABC$ , the point of intersection of the heights). Let us denote the centre of the Euler circle by the point $EOE: EH$. It follows that $EH$ is a common side for the triangles $\Delta AEH$ $\Delta BEH$ and $\Delta CEH$

Let the line $OH$ intersect the sides $AC$ and $AB$ of the triangle $\Delta ABC$ . Then the areas of the triangles $\Delta AEH$ $\Delta BEH$ and $\Delta CEH$ are additive under the condition:

$$S_{BEH} = S_{CEH} + S_{AEH} \tag{6}$$

In this case, the line $OH$ - the Euler line, passes through the point$M$ (centroid) of the triangle, as well as the point $E$ - the centre of the Euler circle $(OE = EH)$ of the triangle $\Delta ABC$ , which is shown in Figure 6. At the same time, $\gamma_O = (O; R = OA)$ characterizes the described circle of the triangle, where $O$ acts as its centre, and $R$ (equal to $OA$ ) is its radius, which is the same for the vertices $AB$ and $C$. Similarly, $\gamma_9 = (E; R_9 = \frac{1}{2}R = EM_1)$ points to a circle of nine points centred on $E$ , where the radius ($R_9$ ) is half the radius of the circumscribed circle, as expressed by the ratio$R_9 = \frac{1}{2}R$ ; where the segment $EM_1$ (where$M_1 = OH \cap AC$ ) is consistent with this value.

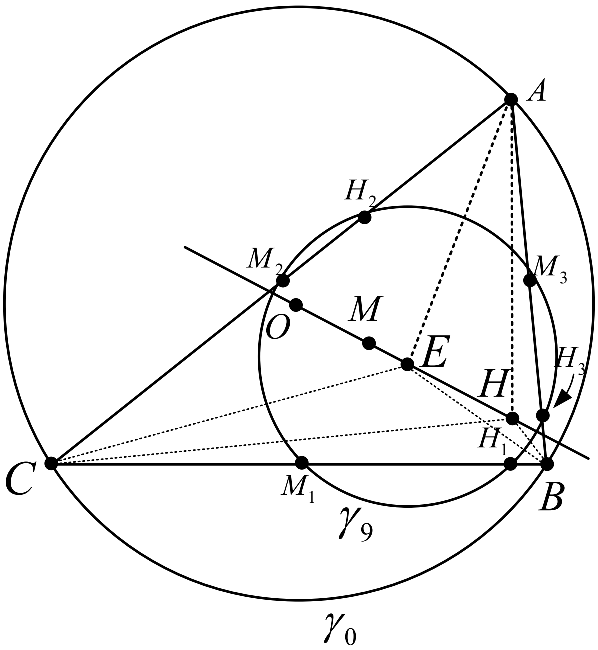


**Figure 6.** Triangle $\Delta ABC$

**Proof**. Let the line $OH$ intersect the sides $AC$ and $AB$ of the triangle $ABC$ . Then the areas of the triangles $AEH$ $BEH$ and $CEH$ are additive under the condition:

$$\rho_A = \rho_B + \rho_C$$

This equality implies that the areas of triangles $\Delta AEH \Delta BEH$ and$\Delta CEH$ have the corresponding ratios, since they all rest on the common base $EH$.

Let $EH$ be the length of the segment (base) of the triangles formed by the vertices $AB$ and $C$ and the point $H$. Then the areas of the triangles are calculated using the following formulas:

For the triangle $\Delta AEH$

$$S_{AEH} = \frac{1}{2} EH * AA_1 = \frac{1}{2} EH * \rho_A$$

For the triangle $\Delta BEH$

$$S_{BEH} = \frac{1}{2} EH * BB_1 = \frac{1}{2} EH * \rho_B$$

For the triangle $\Delta CEH$

$$S_{CEH} = \frac{1}{2} EH * CC_1 = \frac{1}{2} EH * \rho_C$$

Where $EH$ is the length of the base;

$\rho_A, \rho_B, \rho_C$ are the height of the triangle drawn from the vertices $ABC$ to the line $EH$.

Given that the sum of the distances $\rho_B$ and $\rho_C$ is equal to $\rho_A$ , which follows from the geometric properties of the points under consideration, adding the equations for the areas of the triangles $\Delta BEH$ and $\Delta CEH$, we obtain the following equality:

$$S_{BEH} + S_{CEH} = \frac{1}{2} EH * (BB_1 + CC_1) = \frac{1}{2} EH * (\rho_B + \rho_C)$$

Thus, substituting the value of $\rho_A$ into the data, we get the equation:

$$\frac{1}{2}\mathrm{EH} * (\rho_B + \rho_C) = \frac{1}{2}\mathrm{EH} * \rho_A = S_{AEH}$$

The equation proves that the area of $\Delta AEH$ is equal to the sum of the areas of the triangles$\Delta BEH$ and $\Delta CEH$ . In general, it can be represented as follows:

$$S_{BEH} = S_{CEH} + S_{AEH}$$

Thus, the equation will look like this:

$$S_{BEH} + S_{CEH} = S_{AEH} \Rightarrow S_{BEH} = S_{CEH} + S_{AEH}$$

That is, the area of $\Delta BEH$ is equal to the sum of the areas of triangles $\Delta CEH$ and $\Delta AEH$. This demonstrates the additivity of the areas when the line $OH$ passes through the triangle $\Delta ABC$, which is analogous to the previously proven results for the cases when the line passes through the centroid. In this situation, the line $OH$ not only contains the centroid $M$, but also passes through the centre of the Euler circle $E$, for which the condition $OE = EH$ is satisfied.

The additivity of the distances $\rho_{A,}\rho_B$ and $\rho_C$ from the vertices of the triangle to the line $OH$ is proved, which allows us to prove the corresponding area ratios. The connection between the construction of the nine-point circle and the classical centres of the triangle $\Delta ABC$ (the centre of the described circle $O$ , the orthocentre $H$ and the centroid $M$), which lie on the standard Euler line and have known ratios of segments between them, is confirmed.

**Example 3:** Given a geometric structure based on a triangle formed by the centres of the externally inscribed circles for the triangle$\Delta ABC$ . This triangle is denoted by $I_aI_bI_c$ (where $I_aI_bI_c$ are the centres of the externally inscribed circles for triangle $\Delta ABC$ ) has an orthocentre that coincides with the point $I$, which is the point of intersection of the bisectors for triangle $\Delta ABC$ (Figure 7) (Hetmanenko, 2024).

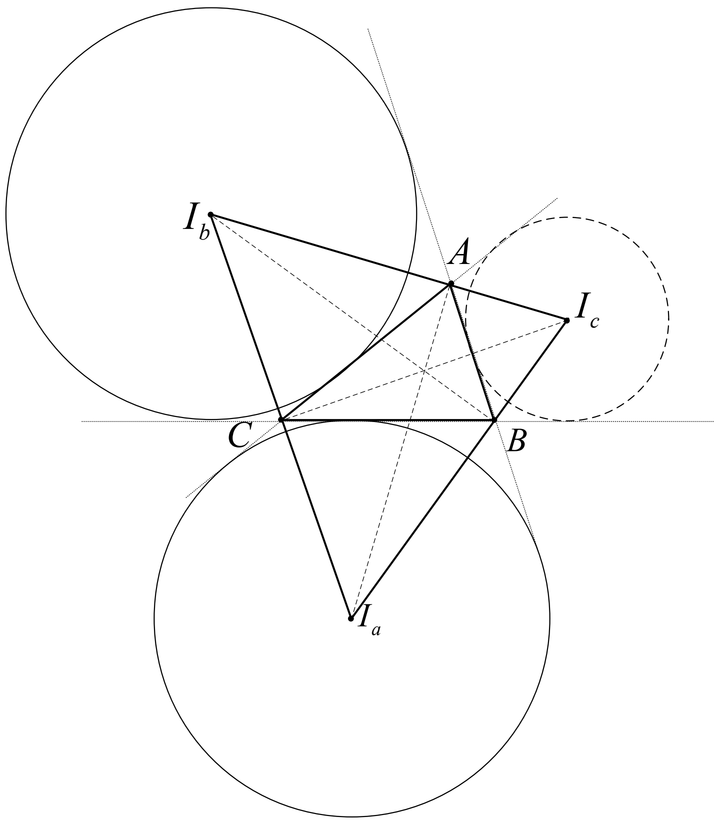


**Figure 7.** The point of Intersection of the Bisectors of a Triangle $\Delta ABC$

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The following conditions apply to triangle $I_aI_bI_c$ :$I_aA \perp I_bI_c$ and $I_bB \perp I_aI_c$ ; that is, the segments $I_aA$ and $I_bB$ are the heights of triangle $I_aI_bI_c$. At the same time, for triangle $\Delta ABC$, the segment $AI_a$ denotes the internal bisector of angle $\angle BAC$; respectively, the segments $BI_a$ and $CI_a$ have similar properties.

The point $O$ is the centre of the circumscribed circle around the triangle $\Delta ABC$ (i.e., the equality $\gamma_O = (O; R = OA)$ ), which is the centre of the Euler circle for the triangle $I_aI_bI_c$ . Triangle $\Delta ABC$ is an orthocentric triangle for triangle $I_aI_bI_c$ , which follows from Figure 8.

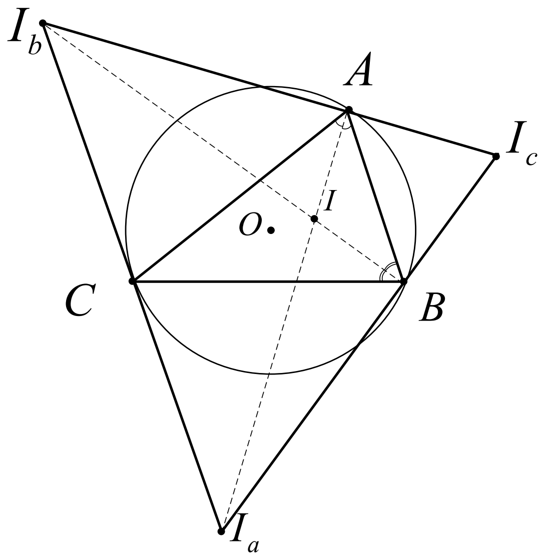


**Figure 8.** Triangle $\Delta ABC$ as an Orthocentric Triangle for Triangle $I_aI_bI_c$

Thus, the circle $\gamma_O$ described around the triangle $\Delta ABC$ is the Euler circle for the triangle $I_aI_bI_c$ . This can be represented by the following inequality: $I_aI_bI_c = I_aI_bI_c$ , which is governed by the relations $O = E$ and $I = H$. That is, in this case, the centre of the described circle of the triangle $\Delta ABC$ coincides with the centre of the circle of nine points (which together are the Eulerian centre) of the triangle $I_aI_bI_c$ , and the pointI acts as the orthocentre.

**Proof**. Consider the analogy to Example 2 with the triangle $I_aI_bI_c$ : Let the line $OI$ intersect the sides $I_aI_b$ and $I_aI_c$ of triangle $I_aI_bI_c$ . Then the areas of triangles $I_aOII_bOI$ and $I_cOI$ are additive. A visual representation of the intersection is shown in Figure 9.

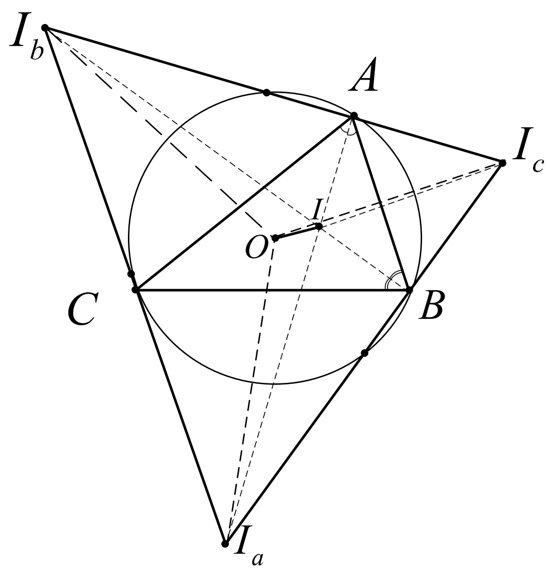


**Figure 9.** Visual Representation of the Intersection in a Triangle $I_aI_bI_c$

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In this case, the line $\mathrm{OI}$ for the triangle $\mathrm{I_aI_bI_c}$ is the Euler line, and therefore passes through the centroid of the triangle $\mathrm{I_aI_bI_c}$ . The segment $\mathrm{OI}$ is a common side for triangles $\mathrm{I_aOII_bOI}$ and $\mathrm{I_cOI}$ ; and the distances from the vertices $\mathrm{I_aI_bI_c}$ of triangle $\mathrm{I_aI_bI_c}$ to line $\mathrm{OI}$ are additive, satisfying the additivity condition, which follows from a general geometric property similar to the previously proven one.

On the basis of the above, we propose to designate the points $\mathrm{X_A}$,$\mathrm{X_B}$, and $\mathrm{X_C}$ in Figure 10 as orthogonal descents from the vertices $\mathrm{I_aI_bI_c}$, respectively, to the line $\mathrm{OI}$ ; that is, the following conditions are met: $\mathrm{I_bX_B\perp OII_cX_C\perp OI}$ and $\mathrm{I_aX_A\perp OI}$ . So, let $\mathrm{I_aX_A} = \rho_1\mathrm{I_bX_B} = \rho_2$ ; and $\mathrm{I_cX_C} = \rho_3$.

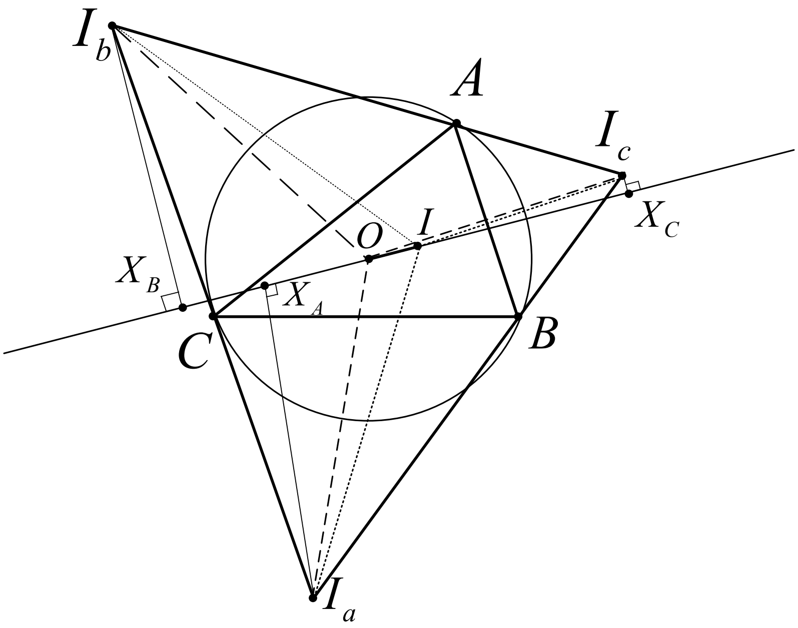



**Figure 10.** Points $\mathrm{X_AX_BX_C}$ as Orthogonal Descents from Vertices $\mathrm{I_aI_bI_c}$ to the Line $\mathrm{OI}$

Adhering to the additivity condition, we obtain the following relationship:

$$\rho_1 + \rho_2 = \rho_3$$

In this case, we will use the classical triangle area formula, which allows us to express the areas of triangles in terms of their heights, which is key to proving the additivity of areas.

For the triangle $\mathrm{S_{I_aOI}}$ , the formula looks like this:

$$\mathrm{S_{I_aOI}} = \frac{1}{2}\mathrm{OI} * \mathrm{I_aX_A} = \frac{1}{2}\mathrm{OI} * \rho_1$$

For the triangle $\mathrm{S_{I_bOI}}$ , a similar formula is used:

$$\mathrm{S_{I_bOI}} = \frac{1}{2}\mathrm{OI} * \mathrm{I_bX_B} = \frac{1}{2}\mathrm{OI} * \rho_2$$

For the triangle $\mathrm{S_{I_cOI}}$ , respectively:

$$\mathrm{S_{I_cOI}} = \frac{1}{2}\mathrm{OI} * \mathrm{I_cX_C} = \frac{1}{2}\mathrm{OI} * \rho_3$$

Given that the sum of the distances $\rho_B$ and $\rho_c$ is equal to $\rho_A$ , which follows from the geometric properties of the points in question, adding the equations for the areas of triangles $\mathrm{S_{I_bOI}}$ and $\mathrm{S_{I_cOI}}$ , we obtain the following equality:

$$\mathrm{S_{I_bOI}} + \mathrm{S_{I_cOI}} = \frac{1}{2}\mathrm{OI} * (\mathrm{I_bX_B} + \mathrm{I_cX_C}) = \frac{1}{2}\mathrm{OI} * (\rho_2 + \rho_3)$$

According to the condition of additivity, the distances from the vertices of the triangle to this line are additive if the following equality holds:

$$I_aX_A = \rho_1 = \rho_2 + \rho_3$$

It follows that:

$$\frac{1}{2}OI * (\rho_2 + \rho_3) = \frac{1}{2}OI * \rho_1 = S_{I_aOI}$$

Thus, the following equality is established:

$$S_{I_bOI} + S_{I_cOI} = S_{I_aOI}$$

Thus, it is proved that the areas of the triangles formed by the intersection of the line $OI$ with the sides of the triangle $I_aI_bI_c$ are additive since the general property of additivity of distances is fulfilled.

These examples explain to students that the considered system of relationships between the classical points of a triangle $\Delta ABC$ makes it possible to obtain additive relations for the distances from the vertices to a given line and for the areas of the parts of the triangle. These conclusions indicate the possibility of creating an infinite number of triangles with the property of additivity of their areas, taking into account the need to fulfil the additivity condition.

### *4.3. The Process of Implementation of the Proposed Methodological Approach in the Process of Teaching the Additivity of the Area of Triangles to Students of Mathematical Specialities*

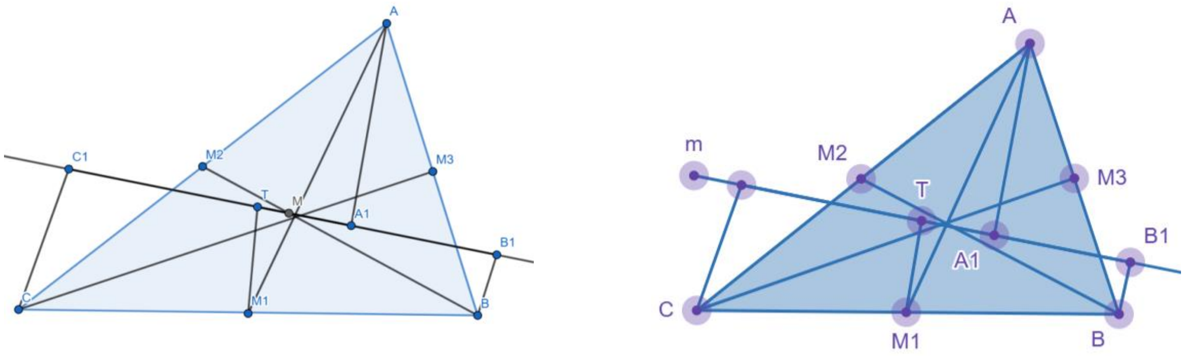
The developed methodological approaches to the study of the additivity of the area of triangles are proposed to be taught using digital technologies, in particular through interactive methods and the creation of visual models. For further analysis of the effectiveness of teaching the topic of additivity of the area of triangles to students of mathematical specialities, we propose to digitize the visual material proposed in the methodological approach. During the literature analysis, the most relevant types of digital platforms were evaluated, in particular GeoGebra and Desmos Geometry (Table 1), which contain the appropriate tools for constructing triangles.

**Table 1.** Comparative Analysis of the Capabilities of GeoGebra and Desmos Geometry for Teaching Geometric Properties of Triangles

| Features | GeoGebra | Desmos Geometry |
|---|---|---|
| Flexibility of parameters | It allows you to change side lengths, angles, and point coordinates in real-time, which facilitates the analysis of dependencies between triangle parameters (Bilousova et al., 2022; Juandi et al., 2021; Machromah et al., 2019). | It contains a convenient mechanism for building shapes, but the ability to vary the parameters of triangles is limited compared to GeoGebra (Aksu & Zengin, 2022). |
| Modelling spatial shapes | It has tools for three-dimensional modelling and analysis of geometric objects in space (Ziatdinov & Valles Jr, 2022). | It is suitable for analyzing the additive properties of shapes, in particular triangles, but without full support for 3D modelling (Aksu & Zengin, 2022; Machado et al., 2023). |
| Analysis of additive properties | It is used to study the area of triangles through variable parameters, which helps to identify patterns in the areas of composite shapes (Bilousova et al., 2022; Dahal et al., 2022; Juandi et al., 2021). | It provides efficient modelling of the additivity of triangle area through integration with graphical tools, which allows comparing the results of different constructions (Aksu & Zengin, 2022; Machado et al., 2023). |
| Process visualisation | It offers ample opportunities for animation and interactive demonstrations of variable geometric parameters (Bilousova et al., 2022; Dahal et al., 2022). | It allows you to quickly build and modify geometric structures, which is suitable for studying the additive properties of triangles (Machado et al., 2023). |
| Ease of use | It has a more complex interface but offers broader possibilities for building and analyzing geometric shapes. | It features a simple and intuitive interface that makes it easy for beginners to use. |
| Digitization of methodological material | It allows the creation of digital models for further use in the educational process (Haciomeroglu et al., 2009; Ziatdinov & Valles Jr, 2022). | It is suitable for interactive methods, including integrating web resources and digital platforms (Aksu & Zengin, 2022). |

Source: compiled by the author based on Aksu & Zengin (2022); Bilousova et al. (2022); Dahal et al., 2022; Haciomeroglu et al. (2009); Juandi et al. (2021); Machado et al. (2023); Machromah et al. (2019); Ziatdinov & Valles Jr (2022).

In order to fully understand the advantages and disadvantages of these digital tools, personal observations and theses known in scientific discourse that indicate the feasibility of using visualization tools in mathematical education were summarised. Current research confirms the effectiveness of using dynamic geometry programmes such as GeoGebra in teaching mathematical concepts, including the additivity of triangle areas (Juandi et al., 2021). When creating their own geometric shapes (Figure 11), the effectiveness of using dynamic geometric programmes in terms of developing spatial thinking was confirmed due to the ability to change the configuration of triangles to test the hypothesis of their area online




**a)** GeoGebra Interactive Toolkit **b)** Desmos Geometry Interactive Toolkit

**Figure 11.** An Example of Using an Interactive Tool to Visualize a Triangle

In addition, changing the parameters of geometric shapes in real time allows students to better understand additivity principles through experimental modelling in GeoGebra (Bilousova et al., 2022). Therefore, it is proposed that the interactive tools of GeoGebra be used to visualize the triangle given earlier in the general case. It should also be noted that Yeung & Ng (2023) point out one of the most controversial drawbacks of the digitalization of mathematics education - students' intuitive perception of the material, which leads to the avoidance of a deeper analytical understanding of the material. In the course of the experimental study, we did not observe an over-reliance on graphical methods, so such visualization does not replace the formal proof of *the additivity of the area of triangles*.

### *4.4. Determining the Effectiveness of the Methodological Approach in Teaching the Additivity of the Area of Triangles to Students of Mathematical Specialities*

In order to understand the applicability of the formed methodological approach and determine the effectiveness of the methodological approach in the process of teaching the additivity of the area of triangles, a study was conducted among students of mathematical specialities. The experimental study includes control and experimental groups, each comprising 33 students. For each group, 2 pairs were conducted, which in the first case contained traditional materials. In the second case, digital visualization tools in GeoGebra were used by our methodological recommendations. The results of the final assessments were analyzed using the Descriptive Statistics tool in the JASP statistical software, as shown in Table 2.

**Table 2.** Results of Descriptive Statistics

| | Control group | Experimental group |
|---|---|---|
| Valid | 33 | 33 |
| Missing | 0 | 0 |
| Mean | 8.091 | 9.455 |
| Std. Deviation | 1.942 | 1.804 |
| Minimum | 4.000 | 6.000 |
| Maximum | 12.000 | 12.000 |

Source: compiled by the author

In the control group, the mean value of the final grades was Mean = 8.091, while in the experimental group the mean score was Mean = 9.455; indicating a higher level of success among students who were taught using the improved methodology, albeit insignificantly. The value of the standard deviation in the control group (Std. Dev. = 1.942) indicates a more significant variability of grades compared to the experimental group (Std. Dev. = 1.804), which, in turn, indicates a relatively more uniform level of learning in the context of digital technologies. The obtained results confirm the expediency of using digital tools in the educational process, which contributes to better learning and reduces the variance of students' performance.

## 5. Discussion

The concept of additivity is central to understanding quantitative relationships in mathematical structures, particularly in geometry. While theoretical research has elaborated on additivity in algebraic and topological contexts (e.g., Aziz et al., 2023; Aziz et al., 2024; Zhang et al., 2018), their educational applicability often remains underexplored. For example, Aziz et al. (2023) generalize Jordan maps on triangular rings that force additivity, and in a subsequent study (Aziz et al., 2024) analyze the process of supporting additive properties by mathematical structures. However, such approaches can be adapted to explain the patterns of triangle areas in the educational process. Damásdi et al. (2019) argue that the additive properties of triangles in combinatorial geometry are investigated, which allows us to estimate their distribution in planar line arrangements. However, in the context of teaching, these approaches often remain abstract and need to be adapted for practical use in the learning process. Particular attention should be paid to the additivity of the arcs of the Euler circle, which, according to Hetmanenko (2023), occurs in any circle described around a triangle, allowing us to consider the area of triangles as a special case of more general geometric property. Pamfilos (2020) analyses triangles with fixed circumscribed circles, determining the maximum area or perimeter configurations. Similarly, the link between affine invariance and additive properties, as studied by Croft et al. (2012), underscores the didactic potential of integrating digital tools to visualize such invariants. Kaufmann (2014) also emphasizes the role of additivity in studying the relationships between various geometric characteristics, particularly in the context of Euler's circle arcs. We believe that using such models in mathematics teaching can contribute not only to a better understanding of additivity by students but also to the overall development of spatial thinking. Thus, our study not only confirms existing theoretical claims but also offers empirical validation for their implementation in educational contexts.

## 6. Conclusion

The obtained results of the experimental study confirm that the use of visualization tools, in particular the GeoGebra and Desmos Geometry platforms, allows not only to improve the quality of learning but also to optimize the processes of teaching mathematical disciplines by reducing the variability of students' performance. It has been found that the use of interactive methods contributes to a more systematic learning of the concept of additivity of the area of triangles, which has further application in the study of the general principles of geometric additivity and related topics, particularly in calculus of variations and mathematical modelling.

The implementation phase included structured lessons using these platforms, where students interacted with dynamically constructed geometric configurations, allowing for immediate visual feedback and a more profound understanding of additive relationships. This integration proved particularly valuable for reinforcing abstract geometric principles through hands-on digital exploration. However, certain limitations should be noted. The study was conducted within a limited sample and timeframe, focusing solely on post-test results without pre-test comparisons. In addition, the long-term impact of digital tools on mathematical thinking remains an open question that requires further research.

## Suggestion

Based on the findings, it is recommended that mathematics curricula at both higher and secondary levels integrate dynamic geometry platforms such as GeoGebra and Desmos to improve students' spatial thinking and deepen their understanding of key concepts, such as domain additivity in geometry. In addition, teacher education programs should include instructional modules focused on the effective use of visualization software, ensuring that teachers are prepared to foster student engagement and promote more profound understanding of mathematics.

Prospects for further research include improving methodological approaches to integrating digital tools into the mathematics teaching process and expanding the empirical base by conducting larger-scale studies aimed at assessing the long-term effect of using such methods in the educational process.

## Declarations

**Author Contributions.** L.H.: Literature review, conceptualization, methodology, data analysis, review-editing and writing, original manuscript preparation, have read and approved the published on the final version of the article.

**Conflicts of Interest.** the author declare no conflict of interest.

**Data Availability Statement.** The data that supports the findings of this study are confidential as bound by the human ethics application.

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