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Assessing the Impact of Competency-Based Integration of Mathematical and Natural Science Knowledge on Critical Thinking, Analytical Skills, and Real-World Problem-Solving

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Abstract

Background: In modern education, there is a growing need to adopt a competency-based approach that supports the integration of knowledge across disciplines. One promising example is the use of Euler's formulas, which fosters comprehensive student development and the practical application of knowledge in real-world contexts. Geometry, particularly when dealing with lesser-known formulas, offers opportunities for creative, non-standardised instruction.

Objective: This study aims to assess the effectiveness of implementing a competency-based approach that integrates mathematical and natural science knowledge, using Euler's formulas, with a focus on developing students' critical thinking, analytical skills, and real-world problem-solving abilities.

Methodology: This study utilised a comparative theoretical analysis of classical proofs for Euler's formulas to identify and extract their untapped pedagogical potential. Using geometric didactic modelling, the researcher developed enhanced, visually intuitive proofs designed to bridge the gap between abstract algebra and spatial geometry. These integrated tasks – combining geometry and physics – were piloted in the “Emotional Formula Geometry” project (Kyiv) and implemented during undergraduate seminars to assess their classroom efficacy. The qualitative validity of the tasks was established through expert peer review, and instructional reliability was ensured through consistent longitudinal use across multiple educational settings to assess student engagement and conceptual retention.

Results: The reinterpreted proofs of Euler's formulas facilitated a clearer understanding of complex geometric relationships. Their integration into practical construction tasks improved student engagement and interdisciplinary thinking.

Conclusion: Tasks based on Euler's formulas effectively promote interdisciplinary connections, enhance students' spatial reasoning, and contribute to improved educational outcomes in mathematics and science.

Unique Contribution: The study introduces didactically enhanced versions of Euler's formulas and demonstrates their practical application in geometry-physics integration. This approach enriches teaching methodology and supports competency-based education.

Key Recommendation: Further research should explore broader applications of classical

mathematical relationships in cross-disciplinary learning scenarios and evaluate their long-term impact on student competency development.

Keywords: first Euler's formula, second Euler's formula, product of chord segments, square of tangent is equal, described circle, exterior circle, Mansion circle, geometric method, algebraic method.

Introduction

In today's rapidly changing world, education is understood not only as a process of knowledge transmission but also as an environment for the development of a personality capable of critical thinking, problem-solving, and adaptation to new challenges. That is why the concept of a competency-based approach, which focuses on the development of key skills rather than merely learning facts, has gained particular importance in education systems worldwide (Adeline, 2024).

One of the most urgent tasks of modern education is to overcome the fragmentation of knowledge accumulated by students within individual disciplines and to create conditions for the formation of a holistic, integrated picture of the world in their minds (Miseliunaite et al., 2022). In this context, interdisciplinary connections play an important role, as they not only expand the educational content but also foster the development of systematic, flexible thinking, which is necessary for navigating the complex and dynamic modern world (Ivanitskaya et al., 2002). Integration of knowledge from different subject areas helps students not only to see the connections between scientific concepts, but also to realise the significance of the knowledge gained in real-life situations. However, in practice, implementing cross-curricular integration often proves challenging for teachers. It requires not only deep knowledge of the educational material but also a strong methodological culture, the ability to engage in pedagogical reflection, and a willingness to creatively search for and adapt teaching strategies to new educational realities (Adeoye et al., 2024).

Of particular value for integrative learning are mathematical concepts that can combine knowledge from different fields. One of these concepts is Euler's formula – a deep, universal equation with applications in physics, computer science, geometry, electronics, and even in the philosophy of beauty (Devlin, 2021). Their use as a tool for interdisciplinary interaction opens up new horizons in building meaningful, logically connected learning (Melnikova, 2017).

Aim of the study

The purpose of this study is to evaluate the effectiveness of integrating mathematical and natural science knowledge through the use of Euler's formulas in developing critical thinking, analytical skills, and the ability to solve real interdisciplinary problems.

Literature review

The competency-based approach is now the key to education, as it helps to clearly define what graduates should know and be able to do. This facilitates the recognition of diplomas, the comparison of qualifications, and the maintenance of academic and professional mobility internationally (Holubnycha et al., 2022).

The current educational system, as the analysis shows, is not yet able to effectively support the reproduction of social capital, a key resource for development in the context of the fourth industrial revolution. To overcome this challenge, it is necessary to develop a substantially revised, legally sound model of competency-based education grounded in a solid academic foundation. The proposed approaches can serve as a practical guide not only for Ukrainian educational reforms but also for European countries with similar cultural and educational traditions (Yuldashev et al., 2022).

The importance of teaching future computer science teachers is substantiated. The application of fuzzy set theory, a tool for modern pedagogical analysis, enables future teachers to conduct flexible assessments of students' competencies, analyse educational methods, and make informed decisions (Papagiannopoulou & Vaiopoulou, 2024). The practical experience of introducing this topic into the course of computer disciplines, the rationale for its teaching, and the positive results of experimental training are presented. Students highly appreciated the relevance and applied value of the knowledge gained, demonstrating their willingness to use fuzzy logic tools in their future professional activities (Bakhov et al., 2021).

The study (Chang & Park, 2021) proposes a method for quantitatively describing architectural forms through their similarity to basic geometric shapes – circle, triangle, and square. Automated models have been developed that deform these shapes, enabling the determination of their similarity using three descriptors: "roundness", "triangularity", and "squareness". Based on this, an integral deformation index was developed to quantify the deviation of the architectural form from the original samples. The method has been tested on the facades and plans of thirty well-known buildings, revealing the dominance of certain geometric features in modern architectural design.

The use of a mathematical model to select a transportation route that maximises the integrated objective function, with the ability to adjust the route at each transportation stage, has proven highly effective for both long-term planning and real-time response (Al-Mutawah et al., 2022). This allows you to quickly change the route and transportation conditions, taking into account real circumstances. Practical testing of the model has revealed risks with an assimilative effect, the realisation of which could significantly alter transportation logistics. The model is an effective tool in transport logistics (Kotenko et al., 2020).

The Euler function plays an important role in the implementation of cryptographic algorithms, particularly RSA, which is used to protect information. An in-depth study of Zn-group properties contributes to improving existing encryption technologies by providing a more precise definition of parameters. (Skuratovskii, 2022).

The practical significance of the Euler function is evident in the RSA algorithm, a cornerstone of information security. Further study of its properties and the structure of Zn groups offers new opportunities to optimise encryption solutions. The study (Derstuganova, 2024a) analysed changes in scientific approaches to interpreting general competencies and conducted a comparative analysis of selected international projects, particularly within the framework of the TUNING initiative. The results provide an analytical basis for assessing the effectiveness of establishing interdisciplinary connections in education, illustrated by the integration of Euler's mathematical formulas into competency-based learning.

The modern educational environment is characterised by a tendency to equate the concepts of "general" and "universal" competencies, indicating a desire to harmonise terminology within higher education across European countries. Such an understanding is essential for establishing interdisciplinary connections within the competence approach, particularly when employing mathematical concepts such as Euler's formula in the context of integrated learning (Derstuganova, 2024b). In this sense, interdisciplinarity is considered the central principle of modern education. It ensures the formation of a holistic system of knowledge, the development of critical thinking, and students' ability to apply the knowledge acquired in real-life situations. As noted by Kotyk et al. (2022), an interdisciplinary approach contributes to increased learning motivation, the development of key competencies, and a deeper, more complex perception of the world.

In the context of the ongoing development of automation and artificial intelligence, everyday life and the professional sphere are continually transforming, and it is precisely human qualities that acquire particular value. Skills for effective interaction, teamwork, leadership, critical thinking, rapid adaptation to new circumstances, and the generation of non-standard solutions are becoming not merely desirable but a key prerequisite for successful activity in an unpredictable, constantly changing future.

Research methods

The study employs a transparent, student-oriented methodology that integrates theoretical analysis, geometric constructions, and practical teaching techniques. The analysis of classical proofs of Euler's formulas aimed to identify the specific points that pose difficulties for students. This enabled the identification of typical barriers to understanding the material and the presentation of it in a more accessible form through step-by-step, visualised proof diagrams.

The interdisciplinary approach was implemented through learning tasks that integrate geometry with physics and demonstrate the practical value of theoretical knowledge. The methodology was tested within the framework of the "Emotional Formula of Geometry" project, which is grounded in active student participation, emotional engagement, and a creative approach to learning. Analysis of learning outcomes and feedback indicated increases in students' understanding of the material, interest in the subject, and confidence in their abilities, confirming the effectiveness of the proposed approach.

Results of the study

Competency-based education is a modern approach that changes the emphasis in higher education. It focuses not so much on the number of courses taken as on what a student really knows, can do and can apply in practice. This format allows consideration of prior experience and acquired skills to progress toward a degree or professional goal more effectively.

The non-standard application of classical mathematical formulas, in particular Euler's formulas, not only enriches the tools for teaching geometry but also opens up new horizons for students to learn and create. The focus is not on the dry technique of calculation, but on understanding the depth of mathematical connections and transforming complex, abstract concepts into accessible and inspiring examples.

In the course of the study, the author's method of integrating mathematical and natural knowledge by applying Euler's formulas to solve problems in constructing a triangle with given geometric characteristics was tested. The application of Euler's formulas (classical relations among the radii of a triangle's circumscribed, inscribed, and exterior circles) enabled an effective integration of analytical and geometric approaches to constructing mathematical models.

This methodology not only improves their proofs but also demonstrates new ways to apply these formulas to construction problems, with both theoretical and practical value. This approach develops spatial imagination and critical thinking and allows students to experience the joy of discovery, an important factor in motivating them to learn. As a result, geometry appears not as a set of rules and formulas, but as a living science that can surprise, fascinate and shape the intellectual potential of young people. To confirm the effectiveness of the proposed approach, two original problems were solved.

Problem 1. Construct a triangle by the centre, the point of intersection of the bisectors and the center of the exterior circle. Point O is the centre of the circumscribed circle around triangle ABC .

Problem condition: AI, AI_a, OI are given. Problem statement: Construct triangle ABC . Basic notation: I is the centre of the triangle, the point of intersection of the bisectors; point I_a is the centre of the circumcircle tangent to side BC ; point O is the centre of the circumcircle around triangle ABC (Figure 1a).

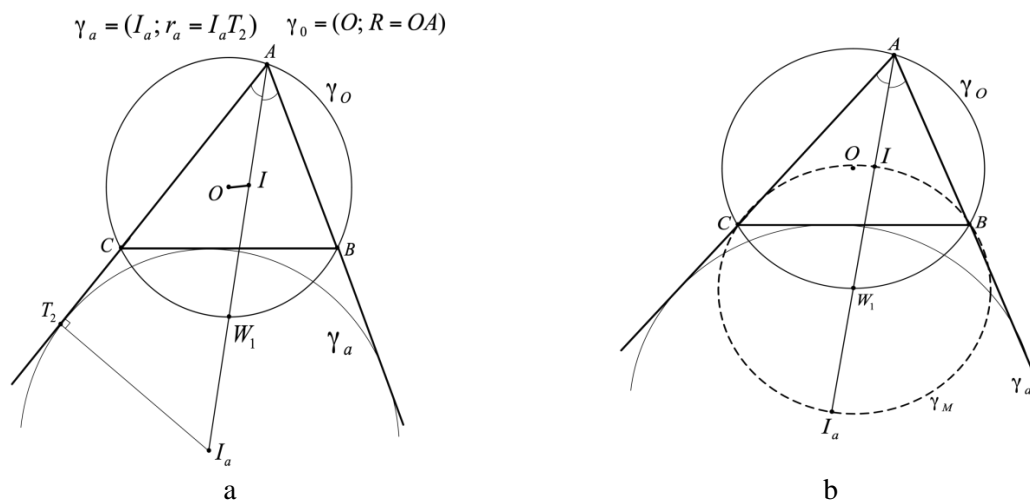
Note: is a circle described around the triangle ABC ;

– Mansion circle;

– is an outer circle touching the party and the continuation BC of parties AC and AB .

Consider the sequence of construction of triangle ABC geometrically:

1. The AI_a bisector of the angle intersects the $\angle BAC$ circumcircle around the triangle at the point O .
2. Triangle AOW_1 is $AO=OW_1=R$ isosceles: are the sides of this triangle.
3. The median perpendicular to the chord AW_1 belongs to the ABC point O , the centre of the circumscribed circle around the triangle .
4. $IW_1=CW_1=BW_1=W_1I_a$ as the radii of the Mansion circle (Figure 1b).



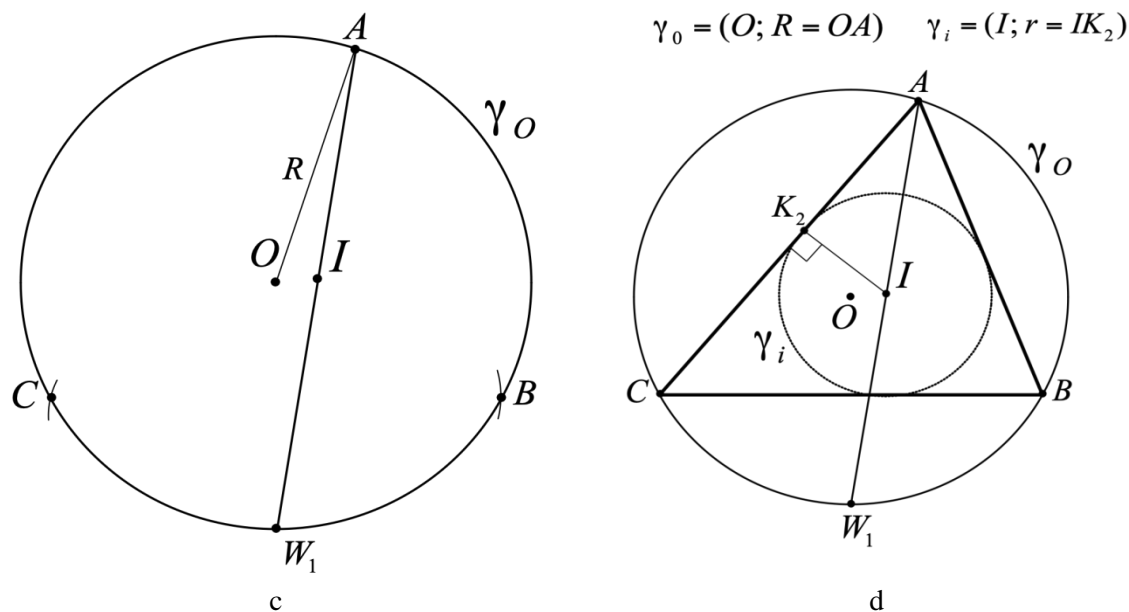


Figure 1. Sequence of construction of a triangle ABC by the incenter, the point of intersection of the bisectors and the center of the exterior circle

Source: compiled from (Kushnir, 1991; Kushnir, 2000;

Note: the segments AI and AI_a are given $IW_1 = W_1I_a$ are the radii of the Mansion circle, where $\gamma = (I; IO)$ is IO the radius.

The construction of triangle ABC will have the following sequence:

1. Construct the segment AI_a (by condition).
2. $I \in AL_\alpha$; $W_1 \in AI_\alpha$
3. Construct the circle γ with centre I and radius OI (Figure 1c).
4. Draw the median perpendicular line t to AW_1 .
5. The line t intersects the circle γ at the point O , the centre of the circumcircle around triangle ABC.
6. The segment $OA(R)$. Let's draw the circle γ_o . ($\gamma_o = (O; R = OA)$; ($\gamma_i = I; r = IK_2$).
7. From the point W_1 , by deflecting the compass IW_1 , we form the points B and C (Figure 1c)

Triangle ABC is constructed.

The second (algebraic) method of constructing a triangle using such segments is based on Euler's first formula.

The first Euler's formula, the distance between the centers of the circumscribed and inscribed circles of a triangle, is as follows (Figure 1d):

$$OI^2 = R^2 - 2Rr. \quad (1)$$

Proof of Euler's first formula (1):

1. From the triangle AIK_2 we get the following equation (see Figure 1d):

$$AI = \frac{r}{\sin \frac{\angle BAC}{2}} \quad (2)$$

From the triangle CDW_1 (Figure 2a):

$$CW_1 = 2R \sin \frac{\angle BAC}{2} \quad (3)$$

where $CW_1 = IW_1$, as the radius of the Mansion circle.

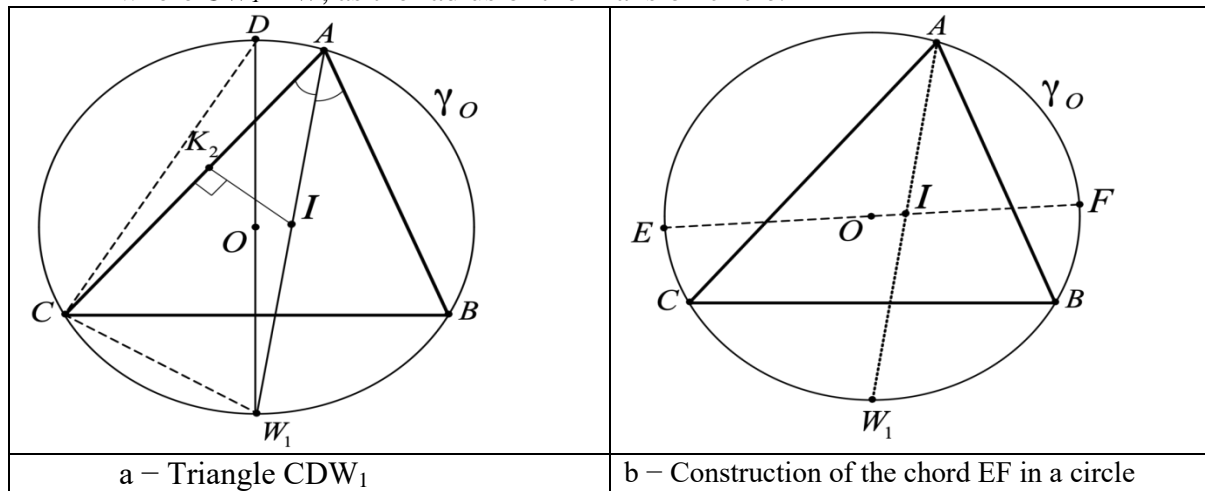


Figure 2. Construction of the chord EF in a circle

Source: built by the authors

The result is equality:

$$AI \cdot IW_1 = \frac{r}{\sin \frac{\angle BAC}{2}} \cdot 2R \sin \frac{\angle BAS}{2} \quad (4)$$

Let's consider the proof of this formula graphically

2. Draw the chord EF (Figure 2b):

$$OI \subset EF$$

$$EI = R + OI \quad IF = R - OI$$

3. $AI \cdot IW_1 = EI \cdot IF$ (as a product of chord segments).

Applying Euler's first formula (1), we obtain the equality:

$$IW_1 = CW_1 = \frac{AI_a - AI}{2} 2Rr = (R + OI)(R - OI) \quad (5)$$

$$2Rr = R^2 - OI^2$$

Otherwise:

$$OI^2 = R^2 - 2Rr$$

Thus, the first Euler formula (1) is proved.

Let's return to the condition of problem 1. Since the lengths of the segments OI , AI and AI_a are known, the first Euler's formula (1) helps us find the radius R of the circumscribed circle around the triangle ABC .

$$If: IW_1 = CW_1 = \frac{AI_a - AI}{2} \text{ (Figure 1a, Figure 1b).}$$

$$If.: IW_1 = CW_1 = 2R \sin \frac{\angle BAC}{2} \text{ (from the triangle } CDW_1 \text{ in Figure 2a).}$$

Substituting the value of $Iph.$ for $IIph.$ and the radius R , we find $\sin \frac{\angle BAC}{2}$, and hence the angle $\angle BAC$

The step-by-step process of building the ABC triangle:

1. Construct the angle $\angle BAC$ and the bisector AI_a .
2. The points I, W_1 belong to the bisector of the angle $\angle BAC$.

Source: compiled by the authors

3. By deflecting the compass IW_1 from the point W_1 we make notches on the sides of the angle $\angle BAC$. We get the points B and C . Triangle ABC is constructed.

In the geometric approach, a set of constructive actions was carried out, in particular: a median perpendicular to the chord of the circle around $\angle ABC$ was constructed, the points were determined using the compass deflection and the properties of the intersection of circles.

The algebraic approach is based on Euler's first formula (1), which enables us to determine the values required to complete the construction.

Task 2. Construct a triangle by the centre, the point of intersection of the bisectors and the centre of the circumscribed circle. Point O is the centre of the circumscribed circle around triangle ABC

Problem condition: given: AI, AI_a, OI_a . The task is to construct the triangle ABC . Key notation: I – the centre of triangle ABC , the point of intersection of the bisectors of this triangle; point I_a – the centre of the circumcircle tangent to side BC of triangle ABC ; point O – the centre of the circumcircle around triangle ABC (Figure 1).

Consider the sequence of the geometric method.

Table 1. Sequence of the geometric method for solving problem 2

Analysis	Construction
$IW_1 = CW_1 = 2R \sin \frac{\angle BAC}{2}$	
<ol style="list-style-type: none"> 1. The bisector AI_a intersects the circumcircle around the triangle ABC at the point W_1. 2. The median perpendicular to the chord AW_1 belongs to the point O – the center of the circumscribed circle around the triangle ABC. 3. $IW_1 = BW_1 = CW_1 = W_1I_a$ as the radii of the Mansion circle, a circle described around the triangle BIC (Figure 2). 	<ol style="list-style-type: none"> 1. Construct the segment AI_a (known by the condition). 2. $I \in AI_a$ $W_1 \in AI_a$ 3. Construct the circle γ_t with center I_a and radius OI_a (Figure 12). 4. To the segment AW_1 draw a median perpendicular – the line t. 5. The line t intersects the circle γ_t at the common point O – the center of the circumscribed circle around the triangle ABC. 6. The segment $AO = R$. Construct the circle γ_o (Figure 1c).

	7. From the point $W_{(I)}$, the compass deflection IW_I forms the points B and C (Figure 1c).
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Source: compiled by the authors

The triangle ABC is built.

The algebraic method of constructing a triangle using such segments is based on Euler's second formula.

A geometric representation of the construction of the segment AI_a is shown in Figure 3a.

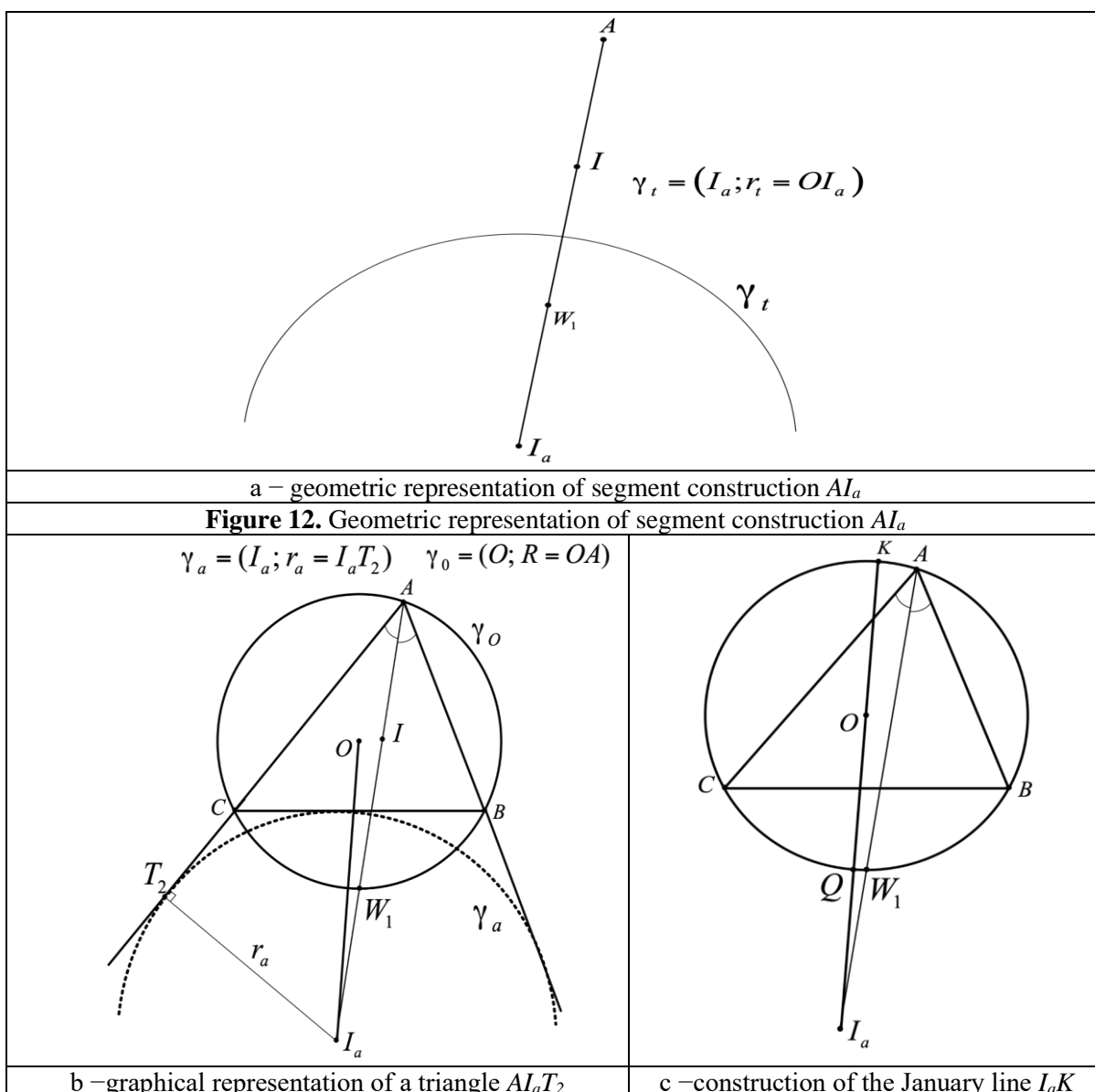


Figure 3. The sequence of constructing a triangle by the incenter, the point of intersection of the bisectors and the center of the circumscribed circle, point O is the center of the circumcircle around triangle ABC

The second Euler's formula, the distance between the centers of the circumscribed circle around the triangle and the exterior circle, is as follows (Figure 3b):

Proof of formula $OI_a^2 = R^2 + 2Rr_a$ (6)

1. From the triangle AI_aT_2 (Figure 3b):

$$AI_a = \frac{r_a}{\sin \frac{\angle BAC}{2}} \quad (7)$$

$$IW_1 = BW_1 = CW_1 = W_1I_a = 2R \sin \frac{\angle BAC}{2} \quad (\text{Figure 8}) \quad (8)$$

Therefore,

$$\begin{aligned} AI_a \cdot W_1I_a &= \frac{r_a}{\sin \frac{\angle BAC}{2}} \cdot 2R \sin \frac{\angle BAC}{2} \\ AI_a \cdot W_1I_a &= 2Rr_a \end{aligned} \quad (9)$$

2. Let's spend January I_aK (Figure 3c): $O \in I_aK$

3. $I_aK \cdot I_aQ = I_aA \cdot I_aW_1$ (the product of January and its outer part) (Figure 3c).

Or,

$$(OI_a + R)(OI_a - R) = AI_a \cdot W_1I_a$$

$$OI_a^2 - R^2 = AI_a \cdot W_1I_a$$

$$OI_a^2 = R^2 + AI_a \cdot W_1I_a$$

Since (10), it is otherwise:

$$OI_a^2 = R^2 + 2Rr_a \quad (10)$$

The second Euler's formula is proved.

Let's return to the condition of problem 2. Since the lengths of the segments OI_a , AI_a and AI_a are known, the second Euler's formula (Δ) helps to find the radius R of the circumscribed circle around the triangle ABC . All subsequent steps for constructing triangle ABC are similar to the algebraic method used in Problem 1.

Advantages of integrating mathematical and natural science knowledge in the educational process

The development of analytical skills is supported by students' ability to structure complex tasks, establish relationships among variables, and select appropriate methods for solving them. The use of Euler's formulas as an integrated concept combines mathematical precision with a deeper understanding of the laws of natural science.

An assessment of work on real cases showed that students who studied using the integration model applied their knowledge more effectively under conditions of uncertainty than the control group. This confirms the effectiveness of the competency-based approach in preparing specialists to solve complex interdisciplinary tasks.

The study analysed the integration potential of geometric problems based on the application of Euler's first and second formulas for the formation of interdisciplinary connections between mathematics (geometry, algebra, trigonometry) and natural sciences (physics, astronomy, technical mechanics). The use of triangle construction problems, taking into account concepts such as the centre, centre of the circumscribed and exterior circle, demonstrated the effectiveness of developing a systematic understanding of the structure of spatial and quantitative relations. The integration of mathematical and natural science knowledge contributes to the development of logical-analytical and spatial thinking, the formation of interdisciplinary modelling skills, a deeper understanding of theory through practice, and increased motivation for research activities.

The integration of mathematics and the natural sciences into the educational process serves as the basis for a holistic worldview for students, enriches interdisciplinary thinking, and enables a better understanding of the significance of the knowledge acquired. This approach expands the understanding of the educational content, strengthens the connection between theoretical provisions and their practical application, and also improves the relevance of the educational experience to the reality of life. It not only activates students' cognitive interest but also develops their ability to critically analyse, effectively solve complex problems, make independent decisions, and take responsibility for the outcomes of their activities, which are the main elements of competency-based learning.

The practice of using Euler's formulas as a tool for cross-curricular integration presents significant challenges, the primary one being the preservation of a rigid division of curricula into separate disciplines. This approach slows down the transfer of educational models and generates several methodological obstacles. As a result, students receive fragmented, non-systemic knowledge that prevents the formation of a holistic view of the world and inhibits the development of systematic and critical thinking. In addition, teachers often face difficulties in applying interdisciplinary approaches – primarily due to the lack of proper training, lack of cross-curricular experience, or limited access to relevant information. The problem is also exacerbated by the lack of clearly formulated methodological recommendations to help teachers make a reasonable choice of subject and achieve an effective balance between mathematical and natural components. This is especially relevant in the context of integrating Euler's formulas, where there is a need to harmonise these approaches with the requirements of current state educational standards. All this slows the transformation of the traditional learning model toward an interdisciplinary, competency-based approach.

Among the main problematic factors, it is necessary to highlight the vagueness of the selection criteria for the essence of integration itself, obstacles to maintaining a methodological balance across different areas of knowledge, and the need to harmonise new didactic approaches with existing state educational standards. These challenges emphasise the need for comprehensive reform of the teacher training system to develop an integrative methodological culture grounded in interdisciplinary thinking, as well as to foster a learning environment that actively stimulates interdisciplinary interaction and the practical application of knowledge in real-world contexts.

Solving the listed problems requires a holistic and systematic strategy for supporting teachers, aimed at the continuous development of professional skills, access to high-quality

interdisciplinary educational materials, and the formation of an open pedagogical space that encourages creativity, experimentation, and the exchange of experience and mutual support. Only under such conditions will knowledge integration become a theoretical concept and a real, effective tool for transforming modern education (Organisation for Economic Co-operation and Development, 2021).

Therefore, the successful implementation of integrated didactic practices, in particular based on Euler's formulas, requires comprehensive professional training for teachers and systematic methodological support that will allow the organic, educationally informed integration of different disciplines and contribute to the formation of a holistic educational space.

Further research should focus on expanding interdisciplinary STEM integration by developing tasks that integrate mathematics, physics, engineering, and digital technologies. It is important to study the impact of this approach on the training of future teachers, the development of systematic and critical thinking, and the comparative effectiveness of integrated and traditional learning. Particular attention should be paid to the creation of digital educational environments that visualise abstract concepts to enhance the applicability and adaptability of education.

Conclusions

The research found that using Euler's formulas as a didactic tool for interdisciplinary integration is an effective means of implementing a competency-based approach in education. An original proof and practical application of the formulas "Distance between the centres of the circumscribed and inscribed circles of a triangle" and "Distance between the centres of the circumscribed and external circles of a triangle" are proposed for new constructive tasks based on geometric and algebraic methods. This approach deepens students' assimilation of educational material, contributes to the development of students' analytical thinking, and fosters skills for establishing logical connections between mathematical models and phenomena in the natural world. At the same time, the research revealed several factors that hinder the widespread implementation of integrated approaches in school practice. Among them are the preservation of the fragmented structure of curricula, the lack of methodological training of teachers for interdisciplinary learning, and the lack of adapted educational materials. These challenges require a systematic, comprehensive solution from educational policy and the pedagogical community to ensure the effectiveness of changes.

Overall, the results of the study confirm the significant potential of integrating Euler's formulas as an effective means of forming key competencies that meet modern educational requirements – in particular, the ability to think critically, analyse information, and creatively approach solving complex problems. The formation of interdisciplinary connections not only enriches the content of learning but also contributes to the development of students' holistic scientific worldview, a prerequisite for quality and humane education in the 21st century.

The integration of mathematical and natural science knowledge, grounded in Euler's formulas, contributes to the development of interdisciplinary thinking and functional literacy in the context of the digitalisation of education.

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