

# Quantitative Estimation of Competency as a Fuzzy Set

Leonid Vasylevych<sup>1</sup> and Ivan Iurty<sup>1</sup>

<sup>1</sup>Borys Grinchenko Kyiv University,  
Department of Information Technology and Mathematical Sciences

lvasilevich@mail.ru, iurty@ukr.net

**Abstract.** The authors of this paper have used the assessment of competence as a fuzzy discrete set consisting of essential capacities. There has been proposed a procedure of competence quantitative estimation on the basis of discrimination index of discrete fuzzy sets fixed on one totality. A linguistic variable “Competency coefficient” has been used here for making appropriate decisions on the grounds of competency quantitative estimation. Assessment of a person’s competency is proposed as a fuzzy discrete set consisting of necessary abilities as its values. Using such competency assessment allows to estimate persons’ competency quantitatively and to compare them.

**Keywords.** Competency-oriented education, competence, capacities, fuzzy discrete set, linguistic variable, membership functions, fuzzification, scalar capacity of any fuzzy discrete set

**Key terms.** MathematicalModelling, MathematicalModel, FormalMethod

## 1 Introduction

The analysis of world education development tendencies demonstrates [1,3] competency-oriented education trend increase. Moreover, competency, which is not only defined by knowledge, abilities, skills but also by considerably greater quantity of factors (coefficients), becomes a major category both in education system and in the job-market. Competency also includes the ability to obtain, to analyze and to revise information; to learn through one’s lifetime; to change in compliance with the job-market demands [1].

Thus, the quantitative estimation of competency necessary for making appropriate decisions is a multicriterial problem, and therefore we need here to derive an integral estimation of competency. Since there is no methodology of working out this problem, it makes the article topical for in it an integral index of a person’s competency coefficient is estimated on the basis of the new competency assessment as a fuzzy discrete set of which essential capacities are values.

Published works analysis. In the work [1], the key competencies concept has been considered and three key competencies have been analyzed (specified by the Organization of Economic Cooperation and Development (OECD) representatives), which are: autonomous activity; interactive facility use; ability to work in socially hetero-

genic groups. Federal Statistics Department of Switzerland and National Center of Education Statistics of the USA and Canada within the program named Definition and Selection of Competencies-- Theoretical and Conceptual principles (“DeSeCo”)” summarized respective scientific results and different countries’ practices. In the work [3] we give a review of works on the topic. But in all those works the qualitative approach to the named subject is solely used, but methods of quantitative competency estimation have never been given.

Thus, the aim of this work is to develop methods of quantitative estimation of competency on the grounds of its assessment as a fuzzy discrete set [2] consisting of essential capacities.

Main results. Competency is defined in UNESCO publications as a combination of knowledge, abilities, values and attitudes used in everyday life. Therefore qualitative assessment of a person’s competency means his (her) ability to perform professional duties or some functions efficiently. But this definition does not give a possibility to estimate expert’s competency on quantitative basis. That is why we proposed to use the following person’s competency definition.

**Definition 1.** A person’s competency is a finite discrete fuzzy set consisting of abilities necessary for a job position or functions necessary for a respective position. Membership functions of the set elements characterize the level of this competency innateness to the person.

**Definition 2.** Abilities are necessary features, characteristics, faculties, qualities, knowledge, techniques, skills and other traits which a person needs to perform duties or functions at a respective position efficiently.

Thereby, in the beginning, we need to define at the discrete set of abilities  $Y = \{y_j : j = \overline{1, m}\}$  membership functions  $\mu_D(y_i) \in [0; 1]$  of the fuzzy set D “Requirements necessary to perform duties or functions at a respective position efficiently”. These membership functions characterize credibility, priority and importance of a respective ability for a respective position or function.

Further we will use the notation of the discrete fuzzy set D in the form [2]:

**Table 3.** Designation D discrete fuzzy set

D=	$y_i$	$y_1$	$y_2$	$y_3$	...	$y_n$
	$\mu_D(y_i)$	$\mu_D(y_1)$	$\mu_D(y_2)$	$\mu_D(y_3)$	...	$\mu_D(y_n)$

or  $D = \langle (y_1 / \mu_D(y_1)); (y_2 / \mu_D(y_2)); (y_3 / \mu_D(y_3)); \dots (y_n / \mu_D(y_n)) \rangle$ .

The set of abilities and respective membership functions will be different for each position. When we specify the Y set we need to apply the Pareto principle, which points that 20% of factors define 80% of the result. In practice, implementation of this principle will lead to the effect that abilities with membership functions less than 0.5 will not be included into the D set. The task of specifying the set of abilities and respective membership functions refers to the task of knowledge estimation by experts and demands creation of respective questionnaires.

As an example, let us specify an IT teacher’s information technology competency in the form of a fuzzy set D:

**Table 4.** Example D representation of discrete fuzzy set

D=	$Y_i$	$y_1$	$y_2$	$y_3$	$y_4$
	$\mu_D(y_i)$	1	0.9	0.7	0.8

in which  $y_1$  - ability to work in Word environment;  $y_2$  is the technique of work in Excel environment;  $y_3$  is the technique of work in Excess environment;  $y_4$  is special software skills (e.g. working out optimization tasks).

To perform a quantitative estimation of a particular teacher’s competency it is necessary to estimate his abilities  $y_i$ . To do so, tests, interviews, exams, respective lessons control and other means can be recommended. Competency grades (their membership functions estimation) can be shown on the scale from 0 to 1. This process is called fuzzification. Hereby, for each person (teacher), we can define in the form of a fuzzy set his personal fuzzy vector of abilities, which defines his competency. To define  $\mu_A(y_i)$  a group of experts can be used who, after analyzing the person, answer the question: “Is ability  $y_i$  attributable to the person?” If the LD expert of L experts give a positive answer, then

$$\mu_A(y_i) = \frac{L_D}{L}. \tag{2}$$

As a rule this question does not have a single-value answer, so, experts can use both binary logic ( $\mu_{A\gamma}(y_i)$  is either 0 or 1, where  $\gamma$  is an expert’s number) and fuzzy logic (multiple-valued verity scale). In so doing they index the value of  $\mu_{A\gamma}(y_i) \in [0; 1]$  (subjective estimate). If quantity of the experts is L, then in the capacity of  $\mu_A(y_i)$  we accept weighted arithmetic mean value of these estimates:

$$\mu_A(y_i) = \frac{\sum_{\gamma=1}^L k_{\gamma} \mu_{A\gamma}(y_i)}{\sum_{\gamma=1}^L k_{\gamma}}, \tag{3}$$

where  $k_{\gamma}$  is the  $\gamma$  expert’s competency estimate.

For quantitative comparison of different persons’ competencies we need, firstly, to compare in pairs finite discrete fuzzy sets D “Demands necessary to perform duties or functions at a particular position efficiently” and  $A_j$  “the j person’s competency” which are specified at one totality Y.

To compare these finite discrete fuzzy sets in pairs it is possible to use the estimate  $P(D, A_j)$  of difference between D and  $A_j$ , which is reduced to the estimate of the traverse of  $\overline{D} \cap A_j$  or  $D \cap \overline{A_j}$  [2]:

$$P(D, A_j) = \frac{|\overline{D} \cup A_j| - |\overline{D}|}{|D|}, \tag{4}$$

where the  $|\dots|$  sign means scalar capacity of any fuzzy discrete set B [2]:

$$|B| = \sum_{x \in X} \mu_B(x) \tag{5}$$

operation  $\overline{B}$  of complementing the fuzzy set B is defined by the membership function [2]

$$\mu_{\overline{B}}(y) = 1 - \mu_B(y), \quad \forall y \in Y \tag{6}$$

operation of two fuzzy sets unification ( $C = B \cup K$ ) has the membership function [2]

$$\mu_C(y) = \max(\mu_B(y); \mu_K(y)), \quad \forall y \in Y. \tag{7}$$

In so doing  $P(D, A_j)$  as a rule is not equal to  $P(A_j, D)$ . This attribute is used to compare fuzzy sets specified at one totality: if  $P(D, A_j) > P(A_j, D)$ , then the fuzzy set  $D < A_j$  and vice-versa.

A person's abilities, which have some membership functions' value greater than the value of respective abilities' membership functions in the D set, must not compensate small values of the  $A_j$  set membership functions. To avoid this it is necessary to perform the  $A_j$  set normalization: membership functions values of the  $A_j$  set which exceed respective values in the D set have to be equated to respective membership functions' values of the D set. Thereby it is necessary to insert the normalized fuzzy set  $A_{jn}$  into the (3) formula.

Let us perform a comparison of two persons' competencies. Let us define one person's competency by means of a fuzzy set  $A_1 = \langle (y_1/0.6); (y_2/0.9); (y_3/0.7); (y_4/0.9) \rangle$  and the other person's competency by means of a fuzzy set  $A_2 = \langle (y_1/0.8); (y_2/1); (y_3/0.5); (y_4/0.9) \rangle$ .

After normalization we have:  $A_{1n} = A_1 = \langle (y_1/0.6); (y_2/0.9); (y_3/0.7); (y_4/0.9) \rangle$ ;

$$A_{2n} = \langle (y_1/0.8); (y_2/0.9); (y_3/0.5); (y_4/0.8) \rangle.$$

The estimate of the difference  $P(D, A_1)$  is equal to (3):

$$P(D, A_1) = \frac{0.6 + 0.9 + 0.7 + 0.9 - 0.6}{1 + 0.9 + 0.7 + 0.9} \approx 0.735.$$

The estimate of the difference  $P(A_1, D)$  is equal to:

$$P(A_1, D) = \frac{1 + 0.9 + 0.7 + 0.8 - 0.9}{0.6 + 0.9 + 0.7 + 0.9} \approx 0.806.$$

We propose to calculate competency coefficient K is as the normalized estimate of differences:

$$K = \frac{\min(P(A, D); P(D, A))}{P(A, D)} \tag{8}$$

This coefficient always belongs to [0;1] interval. If  $P(A,D) > P(D,A)$  then  $K < 1$ , and if  $P(A,D) < P(D,A)$  then  $K = 1$ .

After inserting computed estimates into the (7) formula we have:  

$$K = \frac{\min(0.806; 0.735)}{0.806} \approx 0.912 .$$

Calculation of competence coefficient K for the second person will give values described below:

$$P(D, A_2) = \frac{3.2 - 0.6}{3.4} \approx 0.765; \quad P(A_2, D) = \frac{3.4 - 0.8}{3.2} \approx 0.813;$$

$$K = \frac{\min(P(A, D); P(D, A))}{P(A, D)} = \frac{\min(0.813; 0.765)}{0.813} \approx 0.941.$$

Thereby, we can conclude that the second person's competency is greater than the first one's.

To define a person's competency level basing on the competency coefficient value it is necessary to specify a linguistic variable (LV) [2] "A person's competency coefficient", which we will determine by means of a tuple  $\langle E, E_j, j = \overline{1,5}; \mu_{E_j}(x) \in [0;1]; x = K \in [0;1]; \rangle$ .

Terms of "Competency" LV can be:  $E_1$  – very low competency;  $E_2$  – low competency;  $E_3$  – medium competency;  $E_4$  – high competency;  $E_5$  – very high competency. Trapezoidal membership functions of terms can be defined by experts by means of four numbers  $\langle a : b : c : d \rangle$ , which define each term.

Using trapezoidal membership functions of terms and considering Harrington's scale it is possible to specify "Competency" LV as follows:  $E_1 = \langle 0 : 0 : 0.1 : 0.2 \rangle$ ;  $E_2 = \langle 0.1 : 0.2 : 0.3 : 0.4 \rangle$ ;  $E_3 = \langle 0.3 : 0.4 : 0.6 : 0.7 \rangle$ ;  $E_4 = \langle 0.6 : 0.7 : 0.8 : 0.9 \rangle$ ;  $E_5 = \langle 0.8 : 0.9 : 1 : 1 \rangle$ .

Let estimate  $E_j$  of a term by an  $\gamma$  expert amounts  $\hat{E}_{j\gamma} = \langle a_{j\gamma}; b_{j\gamma}; c_{j\gamma}; d_{j\gamma} \rangle$ , then in the capacity of membership function  $E_j$  of the term we accept a fuzzy quantity

$$E_j = \left\langle \frac{1}{L} \sum_{\gamma=1}^L a_{j\gamma}; \frac{1}{L} \sum_{\gamma=1}^L b_{j\gamma}; \frac{1}{L} \sum_{\gamma=1}^L c_{j\gamma}; \frac{1}{L} \sum_{\gamma=1}^L d_{j\gamma} \right\rangle. \tag{9}$$

To specify terms more appropriately Delphi technique can be applied.

Specifying membership functions lateral branches by straight line segments does not reduce persons' competency estimate's generality but simplifies mathematical operations over fuzzy quantities considerably [4]. In so doing the left  $\mu_l(x)$  and the right  $\mu_r(x)$  lateral branches of the membership linear function have analytical form respectively:

$$\mu_l(x) = \frac{x - a}{b - a}; \quad x \in [a; b], \tag{10}$$

$$\mu_r(x) = \frac{d-x}{d-c}; \quad x \in [c; d]. \quad (11)$$

For the just made example, we have ascertained that the competence coefficient  $K_1=x=0.912$  belongs to  $E_5$  term (very high competency) with membership function (verity) one, and the competence coefficient  $K_2=x=0.88$  belongs to  $E_4$  term (high competence with membership function 0.2 and to  $E_5$  term with membership function 0.8).

Algorithm of a person's competency estimation consists of six stages: the preparatory (1 to 4) and operational (5, 6) ones.

1. Specifying the set of abilities  $Y = \{y_j : j = \overline{1, m}\}$  for a position or functions.
2. Abilities' membership functions assessment (Specifying D fuzzy set) ( (1) and (2) formulae are applied).
3. Ai fuzzy set assessment – “A person's competency”.
4. “A person's competency” LV assessment:  $\langle E, E_j, j = \overline{1, 5}, \mu_{E_j}(x) \in [0; 1], x \in [0; 1] \rangle$ .
5. A person's competency coefficient computing ((3); (4); (5); (6) and (7) formulae are applied).

Computing a person's competency coefficient's membership functions to respective terms LV “A person's competency” (formulae (9) and (10)).

At the preparatory stage experts are used, who define the notion “Competency” as a discrete fuzzy set, the values of which are abilities necessary for a particular position or functions.

Point 3 demands creating respective techniques, tests, problems and tasks that allow estimating various abilities of a person (to find membership functions of each ability).

At the stage of receiving a person's competency quantitative estimate points 5 and 6 are performed.

To specify A fuzzy set of an expert's antecedent characteristics the expert's questionnaire data, his (her) tests, interviews can be used.

The examined competency estimation methodology based upon using fuzzy sets and a linguistic variable allows resolving several problems: conversion from current qualitative competency assessments to quantitative estimation; multicriteriality of competency estimation problem; impossibility of quantitative measuring certain particular indexes of competency; impossibility of real experiments to estimate different persons' competency.

## 2 Conclusions

1. Assessment of a person's competency is proposed as a fuzzy discrete set consisting of necessary abilities as its values. Using such competency assessment allows to estimate persons' competency quantitatively and to compare them.
2. A methodology of a person's competency quantitative estimation is proposed.

3. It is proposed to estimate quantitatively persons' competency on the basis of difference coefficient of finite discrete fuzzy sets  $D$  "Demands necessary to perform duties or functions at a particular position efficiently".
4. It is proposed to specify an expert's competency coefficient in the form of linguistic variable "Competency".

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