# GeoGebra Classroom as a Component for the ICT support of Inquiry-based Mathematics Education in Blended Learning

Mariia Astafieva [1], Oksana Hlushak [1] and Oksana Lytvyn [1]

[1] *Borys Grinchenko Kyiv University, 18/2 Bulvarno-Kudriavska Str, Kyiv, 04053, Ukraine*

**Abstract**

The article describes the essence of pedagogical strategy the Inquiry-based learning (IBL). It considers the features of the pedagogical strategy IBL application in the educational process of higher education institutions during teaching of mathematical disciplines (Inquiry-based mathematics education, abbr. IBME). There are indicated some difficulties to implement the IBME paradigm in the context of blended learning, which limits direct "live" contact between the teacher and students, and students among themselves. Modern techniques are able to compensate "live" communication sufficiently, moreover it is not lost completely due to the blended learning. The most important task is the search for the digital tools that would help implement the main idea of IBME. It is active and research-oriented teaching of mathematics on a constructivist basis, which optimally combines individual (autonomous) and group work of students. Possibilities are revealed, and the expediency of using the virtual platform GeoGebra Classroom in IBME classes is argued. In particular, this expediency is justified to ensure the research orientation of teaching higher mathematics on the basis of inquiry. And also for effective pedagogical control of students individual work and team work developing their own knowledge. In the article there are stated examples of practical exercises in the disciplines "Mathematical analysis" and "Projective geometry and image methods", which illustrate the way how not to lose the opportunity to form ability of students to learn independently through implementing the principle of collaborative learning.

**Keywords**

Higher mathematics education, IBL, IBME, ICT support, GeoGebra Classroom, Blended learning.

## 1. Introduction

Modern trends of human civilization development, fundamental changes in the socio-economic structure of society, scientific and technological progress, rapid technological development, new challenges and threats (i.e. economic, environmental, political, military) require now and highly demand in the future specialists for high-tech industries. This reasons significantly raise requirements level for mathematical training of university graduates. Therefore, the intensification of targeted scientific research, international projects on the issues of mathematical training of students, the development of practices, the study and creative implementation of positive experience is noticeable. In particular, there is an acute issue to change the educational paradigm, such as: leave traditional teaching, when the teacher as a person who provides ready-made facts, to one in which the student is an active recipient of knowledge. The Council of Europe Education Department defines active, research-oriented learning as a leading educational methodology that will help shape the key competences required for a culture of democracy [1].

One of the constructivist learning strategies is Inquiry Based Learning (IBL). Its origins are traced back to the works of Piaget, Dewey, Vygotsky, and Freire. Learning through personal experience, active action and interaction is the main principle of constructivism. IBL is research-oriented learning,

ICTERI-2021, Vol I: Main Conference, PhD Symposium, Posters and Demonstrations, September 28 – October 2, 2021, Kherson, Ukraine
EMAIL: m.astafieva@kubg.edu.ua (M. Astafieva); o.hlushak@kubg.edu.ua (O. Hlushak); o.lytvyn@kubg.edu.ua (O. Lytvyn)
ORCID: 0000-0002-2198-4614 (M. Astafieva); 0000-0001-9849-1140 (O.Hlushak); 0000-0002-5118-1003 (O. Lytvyn)

the process of knowledge constructing knowledge by formulating questions and finding answers to them. The IBL approach, launched in the 1960s by Joseph Schwab, is developed actively in foreign countries. However, in Ukraine only during the last few years it has been included in pedagogical practice, mainly school, as the implementation point of the Concept of the New Ukrainian School (2016) [2].

The inquiry based mathematics education (IBME) is based on the paradigm of teaching higher mathematics, in which students are encouraged to work in the same way as professional mathematicians, such as: observe, experiment, notice properties and patterns, ask questions and look for mathematical and scientific ways to answer them, express hypotheses, summarize, interpret and critically evaluate solutions, communicate effectively in the process of finding, presenting and discussing ways and results. Within this paradigm the teacher is a partner (tutor, facilitator) of overall activities with students. Conditions of distance learning create known problems together with the implementation of such cooperation. It is due to the lack of direct contact between the teacher and students and students among themselves. It is only the use of information and communication technologies and the simplest technical means that helps to ensure information and reproductive training without any special losses, such as: you can post relevant material on the network, record video lectures and the process of solving typical tasks, give similar tasks for independent work, check the correctness of the implementation and set estimates. But IBME needs a fundamentally different model of organization of the educational process in the classroom, when the teacher organizes pedagogical support for independent student learning, student is on teacher's radar, and teacher can effectively guide the trajectory of student's research. So, it leads to new knowledge for student. Therefore, the purpose of our study was to explore the possibilities GeoGebra tools to support such teaching of mathematics (in offline or online formats) and to develop a methodology for their usage.

## 2. Literature Review

There are many characteristics and explanations of the Inquiry based learning essence in scientific sources [3-10]. Summarizing the developments of the researchers, we note that the introduction of research-oriented learning in the educational process in mathematics allows students to be attracted to the real scientific process of discovery, active student participation and responsibility of the person for obtaining new knowledge, promotes deep exposure of students in the process of mathematical knowledge and collaborative work to analyse mathematical ideas.

The use of ICT contributes to an increase in the effectiveness of such training, as noted by researchers Rocha Fernandes, Geraldo W., Rodrigues, António M., Rosa Ferreira, Carlos Alberto [11]. However, they point out that there is not enough research on strategy and didactics on the practical use of ICTs to support IBL teaching in mathematics and science.

## 3. Research Methods

In a course of the research, the following methods were used: analysis of scientific literature, determination of categorical-conceptual apparatus; synthesis, generalization, systematization; diagnostics (interviews and questionnaires).

## 4. Research Results

There are three levels of student research work organization on the way to new knowledge, such as: structured inquiry, guided inquiry, open inquiry [12].

We will describe the activities of teachers and students at each level in the implementation of IBME.

**Level I.** Structured Inquiry

The teacher formulates a research question (task), briefly describes the procedure (solution idea), and students through implementing the proposed procedure or idea come to an answer (get a result). Decision steps, intermediate conclusions or results are explained and argued by students.

**Level II.** Guided Inquiry

The teacher formulates only a research question or task. Students analyze independently (i.e. a known problem or a new one, problem parameters, etc.), formulate hypotheses regarding the idea of a solution or result, choose a way, method, procedure, tools that will help find an answer to the posed question, present and justify their choice and the result, formulate certain conclusions (e.g. either the solution is the only one, or is sensitive to minor changes in parameters, has interesting special cases; or the rational method is chosen, what are its "pros" and "cons", which other solutions are possible, etc.).

**Level III.** Open Inquiry

Students determine independently a research question or task, look for a way to research (solution), develop and implement an appropriate procedure, choose the necessary tools, present their results and conclusions based on the obtained research. Depending on the situation, the teacher acts as a consultant, partner, team member, tutor, facilitator or opponent.

Depending on the level of students readiness and their preliminary experience in search and research activities, the teacher implements one or another level of Inquiry. Obviously, in junior courses one should start from the first or second levels, gradually moving to the third level when the research work of students fully repeats the work of scientists, except the example of solving simple (for a scientist but not for a student) problems.

Our experience shows that the tools of the free dynamic GeoGebra environment are good support for IBME, especially in a blended learning. Stated blended learning we mean a purposeful process of mastering knowledge, skills and abilities, which is carried out by an educational institution within the framework of formal education, and part of which is implemented remotely using digital technologies and computer support.

Research activity in mathematics is impossible without conducting an experiment, mostly imaginary, without a high level of thinking. Especially the so-called "visual" thinking that is, thinking with visual images and with the help of visual operations. GeoGebra has many possibilities to provide the appropriate conditions [13].

Learning through question formulation and research through collaboration, communication and teamwork is a special feature of the IBL [14]. Working on a common task in a micro group, students learn to interact to achieve a common goal. They become resources of each other on the way to new knowledge, and also they learn to reason and hear the arguments of others, to oppose convincingly and improve their thinking in general. However, in order to succeed in the study of mathematics, it is necessary to be able to work autonomously and self-controlled without the help of others. Group work without purposeful and well-founded pedagogical support can lead to a situation when individual students (for example, with low academic achievements) cannot (or do not want to) interact. Obviously, these students will not get any benefit from such group work, they simply "drop out" of the process. They need individual guidance of the self-study process from the teacher's side. The GeoGebra Classroom helps to provide such guidance.

To implement IBME, GeoGebra Classroom can be a tool to engage students in the research process and a means of studying educational material [15]. With GeoGebra classroom teacher can create teaching materials that include: text, video, GeoGebra applet (either previously created or empty), pdf file, image, question (open or multiple choice), and page links. During the GeoGebra classroom, the teacher observes the implementation of the proposed tasks. It is very important that the teacher sees the whole process of group activity and progress of each individual student continuously, not in fragments, so he can quickly respond to mistakes made by the student, adjust his learning or research trajectory, provide timely support, help to understand and eliminate obstacles. Such individual work with each student in the GeoGebra classroom is effective both in distance learning (using video conferencing services) and in classroom. Because even if the class is held in the classroom, the teacher cannot see at the same time what each of the students is doing in the notebook or on the computer. This is possible with the GeoGebra classroom. This tool promotes the implementation of an individual approach, taking into account the characteristics of students.

Each of the students of the group during the GeoGebra classroom does not see what his classmates are doing, so the students work at their own pace, in a comfortable environment. In addition, students cannot "peek" at the idea of a solution from classmates, thus effectively eliminating the possibility of writing off.

The implementation of IBME using GeoGebra class in the educational process is illustrated by two examples of classes in mathematical disciplines for students majoring in 111 "Mathematics".

**Case 1.** In one of the classes in the study of the derivative (academic discipline "Mathematical analysis"), first-year mathematics students were offered two options for the problem (Figure 1): according to the known graph of the function f(x) (option 1) or the derivative f'(x) option 2) to construct a sketch of the graph of the derivative f'(x) or the function f(x) respectively.

Taking into account that first-year students do not have significant research experience, the teacher chose the first level of work organization of structured inquiry. Therefore, the task was accompanied by some instructions from the teacher:

*"Justify the construction by giving reasoned answers to the questions:*

*a) what properties of a given function at certain intervals determine the corresponding (which ones?) properties of the function whose graph we are plotting?*

*b) is there enough information in some parts of the graph of this function to unambiguously depict the graph of the new function? in case of a negative answer, indicate what minimum information is missing.*

*For convenience, build each section of the graph in a different color"*

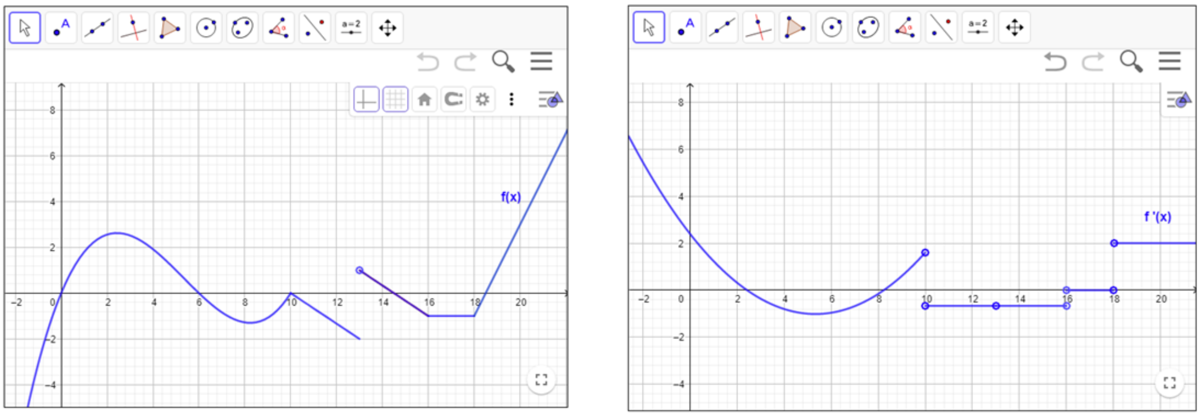


**Figure 1:** Graphs of functions: *f(x)* (left); *f'(x)* (right)

In addition to plotting graphs, students had one more task:

*"Suggest two questions that you would like (or might ask) ask concerning the graph of the function f(x) (f'(x)) about the function f'(x) (f(x))".*

Students performed the construction of appropriate schedules in GeoGebra Classroom. One half on the known graph of the function built a graph of its derivative. On the contrary, the other one, on the graph of the derivative function restored the graph of the function itself. Each student saw only his task and did not know what task his classmates had.

The goal set by the teacher was to form students' ability to consider and to explain mathematical structures (in this case, the derivative of a function) from different points of view, to interpret them in different forms, in particular, analytical and graphic. The achievement of this goal was facilitated by the use of IBME technology and GeoGebra Classroom tools. IBME technology has activated such cognitive processes as asking your own questions, analyzing visual images, the ability to notice details, translating visual information into analytical language and vice versa, argumentation, proof. GeoGebra Classroom allowed each student to independently analyze, draw conclusions and make decisions on constructions, while working at their own pace. The teacher had the opportunity to see all students at once, to observe "online" every step of each student, his/her searches and hesitations, for example, when a student built a certain fragment of the graph, and then replaced it with another one. Thus, according to the student's actions, the teacher could "read" the course of his reasoning, see all the weaknesses and errors, and thus accurately diagnose the problem and take appropriate measures to solve it.

After the graphs were built, the teacher organized a panel discussion of the results. For analysis, the teacher chose two constructed graphs from each option and demonstrated them on the screen (Figure 2, 3).

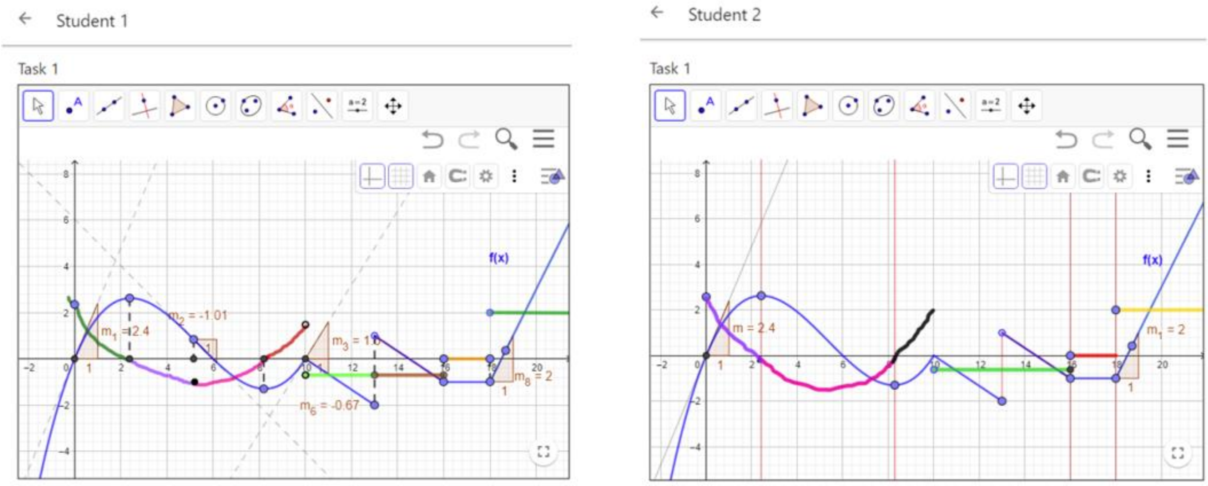


**Figure 2:** Plotting a derivative of a function according to the graph of the function itself

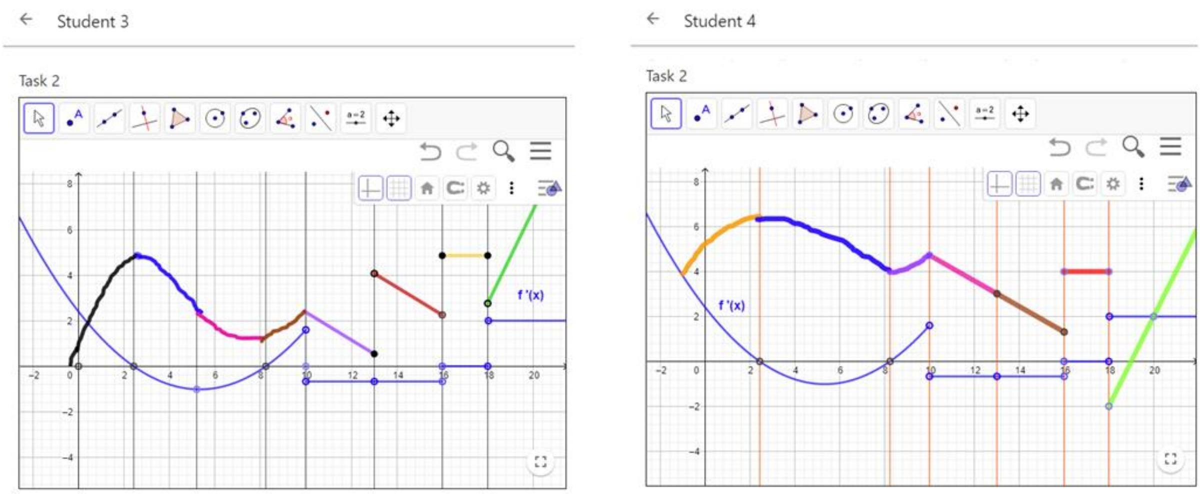


**Figure 3:** Plotting a function from the graph of its derivative

During the discussion, students found out whose graph is closer to the "original", where mistakes were made, their reasons (ignorance or not taking into account which mathematical fact caused the error), explained and argued the construction. In particular, they noted that Student 1 acted more "professionally" than Student 2. Because at all key points of change in the behavior of the graph of the function followed the slope, using the appropriate tool in GeoGebra, which is reflected in the graph of the derivative. Student 2 made most of the transitions "by feel", and also did not reflect the correspondence between the direction of the graph convexity of the function (the sign of the second derivative) and the nature of the monotonicity of its derivative and did not see some breakpoints of the derivative.

Analyzing Student 3 and Student 4 graphs, it was noticed that the students "detected" errors on graph of Student 4 and even explained the reason why both graphs differ not only from each other, but also from the "original". This situation is natural, because the problem of recovering a function from its derivative has many solutions.

The purpose of the second task was to teach students to ask mathematical questions as well as to develop language competence, namely – to formulate them competently in writing (students entered the answers in the appropriate form in the class task).

The questions that were posed by the students have varying complexity level and different levels of research focus. For example, questions whose answers require the direct use of known facts:

1. *How many roots does the equation $f'(x) = 0$ have? If so, what are the roots?*
2. *At what interval does the function $f(x)$ become?*

As well as more complex, which involve research:

3. *Is it possible to specify the intervals of monotonicity of the function $f'(x)$? If so, what are the intervals? On which of them $f'(x)$ increases, and on which it decreases? If not, explain why.*
4. *Does the equation have roots $f'(x) = 0$. If yes, then indicate them.*

Finally, students reflected on what they had learned, evaluated their own progress and learning strategy. The are some statements from students below.

"At first I couldn't figure out what to do, but the teacher's support and explanation helped me more or less cope."

"I liked looking for the slope of a curve in GeoGebra. It's very convenient. I didn't have to use this tool before. But it was difficult to draw with a mouse; a pen would be much easier."

"For some reason it did not occur to me that I should also look at the second derivative. Now, after we have considered it, I understand."

"It was difficult. It's easier when we solve it in a group, because if you don't know something there's someone who knows. But when you do it yourself, you'll certainly understand and won't forget."

**Case 2.** It is known that when solving problems requiring constructions, students encounter difficulties associated with spatial imagination and manipulation of geometric objects within the framework of a flat image of spatial figures. At the same time, it is important that the student must carry out the complex construction on his own, consciously going through the entire algorithm. The problem is that the teacher cannot follow the entire process of building each student in the limited time of the lesson.

The cross-sectional tasks are special here: the images constructed by students during the execution of the same task will be different. As an example, we will demonstrate the use of a classroom for constructing a cross-section of a cylinder using the trace method. First, the teacher asks the students to find out what is the difference between the construction of cross-sections of polyhedra and solids of revolution? Taking into account, we construct cross-sections of solids of revolution by points, this in itself is a long process with a large number of constructions which students perform each at their own pace. The students were given the task: 1) to construct a section of a cylinder with a plane given by a trace on the plane of the lower base and a point on the visible part of the cylindrical surface; 2) analyze the shape of the formed section depending on the position of the cut plane trace relative to the base of the cylinder and a point on the surface of the cylinder. The teacher used a guided inquiry, so she did not give any more instructions and only observed the students' work and "gently" guided them in the right direction.

The group of second-year students majoring in mathematics, who participated in the experiment, was small – 8 students. This allowed the teacher to follow the progress of each student. In Figure 4 shows a screenshot of the Classroom overview.

The figure shows how different the pace of the students' constructions is: at the moment when student 1 completed all the constructions and received a cross-section, Students 6 and 7 had just figured out the problem and started working. Obviously, working together Students 1 and 7 (for example, in pairs) will not help the development of Student 7 and will slow down the development of Student 1. GeoGebra Classroom functionality allowed to the teacher to see in real time everyone's progress or problems and, accordingly, to guide his activities.

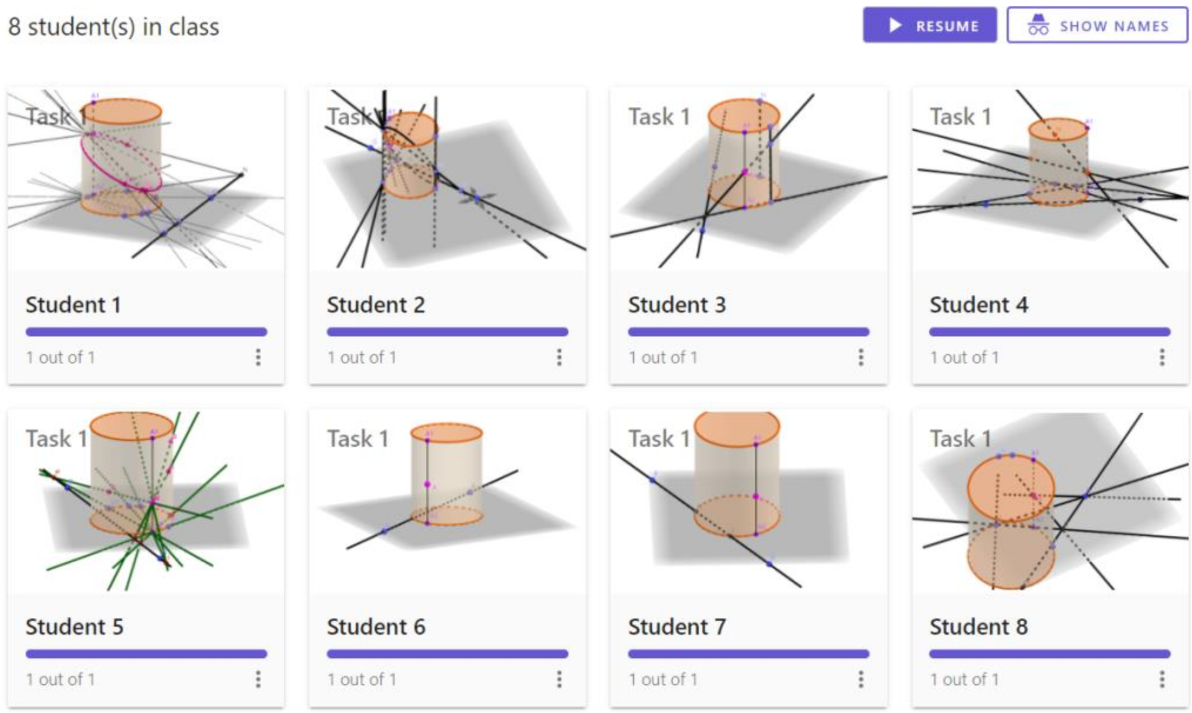


**Figure 4:** Screenshot of Classroom overview "Cylinder cross-section"

In Figure 5 shows the student's attempts to investigate the resulting construction (item 2 of the assignment). All students independently came to the conclusion that the shape of the section of the cylinder changes depending on the position of the trace of the secant plane and the position of point A on the lateral surface of the cylinder. And after a joint discussion the results, under the guidance of the teacher, were formulated a conclusion about all possible cross-sectional options (empty set, point, line, rectangle, ellipse, circle, complex figure of segments and arcs of an ellipse) depending on the position of the secant plane.

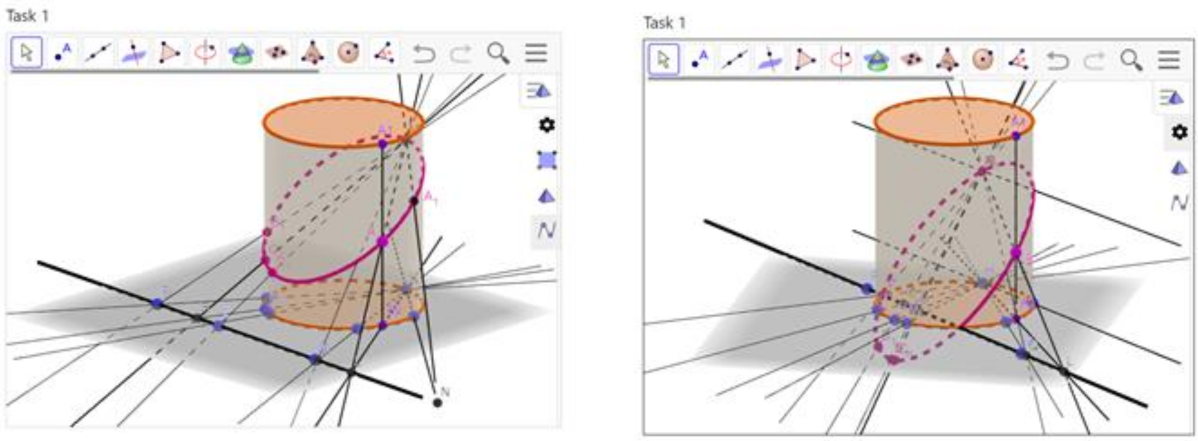


**Figure 5:** Section view depending on the position of the secant plane

To assess the effectiveness of the lesson, a survey of students was conducted. The analysis of their answers showed that the students positively assessed the classes and unanimously recommend using such an approach to those topics of the discipline "Projective geometry and methods of images", which provide for construction tasks. 75% of respondents unequivocally agreed that classes in this format provided a better understanding of the construction algorithm. No one respondent thinks that when working in a notebook it was clearer. Half of the students indicated as the advantages of completing tasks in the classroom that the teacher can immediately suggest if something is being done incorrectly. Three students out of eight indicated that the mistake can be easily corrected. 12.5% of respondents noted both options. The disadvantage of using the GeoGebra environment and working in the classroom is that a quarter of students believe that they cannot compare their building process with the construction in other students, a quarter of students spend more time on the task, and 37.5% of respondents did not see any deficiency at all. Impressions of students after the lesson:

*"I enjoyed working in this format"*

*"Cool and convenient"*

*"Great program, but it takes more practice to navigate easily"*

*"It shows and proves the properties of the cross section. I liked it, but I need to understand the GeoGebra tools"*

*"Classes are interesting, but it is not always clear how this program works"*

*"It was more interesting, so the material was easier to learn"*

## 5. Conclusions and prospects for further research

Analysis of the scientific literature and the results of European research projects and own practice show that IBME's strategy in higher education is effective for training professionals who can use mathematical knowledge and skills to solve professional problems, ready to quickly and effectively acquire knowledge and solve problems as well as autonomously and in a team.

In the conditions of distance and blended learning, which is especially important now, the implementation of such a strategy requires serious ICT support. However, the method of effective involvement of digital tools at IBME is insufficiently developed. This pedagogical technology involves research orientation of individual and team work of students to construct their own knowledge and effective pedagogical guidance of this process, so it requires specific digital tools.

As our case shows, one of such tool is the virtual platform GeoGebra Classroom. It provides a continuous and effective relationship between the teacher and each student, the ability to work both autonomously and in a microgroup, work with interactive mathematical tools, promotes discussion of ideas, and results etc.

We see the prospect of further research in the conduct of a pedagogical experiment on the use of GeoGebra Classroom and the theoretical substantiation of its effectiveness to support experiment-oriented strategies for teaching higher mathematics. At the practical level, attention will be focused on didactic developments in the disciplines of higher mathematics and methodological recommendations for their implementation in the educational process of higher educational institutions.

## 6. Acknowledgements

The research findings of this article were conducted within the framework of the "Partnership for Learning and Teaching in University Mathematics" (PLATINUM) program of the EU Erasmus + KA203 – Strategic Partnership for Higher Education, 2018-1-NO01-KA203-038887. This article reflects only the views of the authors, and the European Commission cannot be held responsible for any use which may be made of the information contained therein.

## 7. References

[1] Competences for democratic culture. Living together as equals in culturally diverse democratic societies. Council of Europe, 2016. URL: https://rm.coe.int/16806ccc07.

[2] The new Ukrainian school. Conceptual principles of secondary school reform, 2016. URL: https://mon.gov.ua/storage/app/media/zagalna%20serednya/Book-ENG.pdf.

[3] C. Manoli, M. Pedaste, M. Mäeots, L. Siiman, T. Jong, et al: Phases of inquiry-based learning: Definitions and the inquiry cycle. Educational Research Review, Elsevier 14, (2015): 47-61.

[4] J. Boaler, Open and closed mathematics: Student experiences and understandings. Journal for Research in Mathematics Education 29(1), (1998): 41-62.

[5] E. Burger, M. Starbird, The Heart of Mathematics: An Invitation to Effective Thinking, Key College Publishing, 2005.
[6] S. Goodchild, A.B. Fuglestad, B. Jaworski, Critical alignment in inquiry-based practice in developing mathematics teaching – Educational Studies in Mathematics 84(3), (2013): 393-412.
[7] S. L. Laursen, C. Rasmussen, I on the prize: Inquiry approaches in undergraduate mathematics. International Journal of Research in Undergraduate Mathematics Education 5(1), (2019): 129-146.
[8] A. Schinck-Mikel, C. Poly, San L. Obispo, The road to present day inquiry-based learning, 2021. URL: http://www.inquirybasedlearning.org/why-use-ibl.
[9] J. Moschkovich, An introduction to examining everyday and academic mathematical practices. In M. E. Brenner & J. N. Moschkovich (Eds.), Everyday and academic mathematics in the classroom (JRME Monograph Number 11*)*. Reston, VA: National Council of Teachers of Mathematics, 2002, pp. 1-11.
[10] C. Rasmussen, M. Zandieh, K. King, A. Teppo, Advancing mathematical activity: A view of advanced mathematical thinking. Mathematical Thinking and Learning 7, (2005): 51-73.
[11] Rocha Fernandes, Geraldo W., Rodrigues, António M., Rosa Ferreira, Carlos Alberto: Using ICT in Inquiry-Based Science Education. Springer International Publishing, 2019.
[12] M. Zion, R. Mendelovici, Moving from structured to open inquiry: Challenges and limits. Science Education International 23 (4), (2012): 383-399.
[13] M. Astafieva, D. Bodnenko, O. Lytvyn, V. Proshkin, The Use of Digital Visualization Tools to Form Mathematical Competence of Students. ICTERI 2020, (2020): 416-422.
[14] B. Jaworski, Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. Journal of mathematics teacher education 9 (2), (2006): 187-211.
[15] J. Olsson, GeoGebra, Enhancing Creative Mathematical Reasoning. Umea, Umea universitet, 2017.