

Methodology of formation of modeling skills through GeoGebra cloud service based on a constructive approach

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Abstract

The authors suggest one of the possible ways to use GeoGebra cloud service. The chosen way concerns the formation of skills to model mathematical objects based on a constructive approach. It is established that the main idea of a constructive approach in the study of Geometry is that acquaintance with the concepts properties begins with constructive activities for their "discovery" and assimilation with a gradual transition to definitions and logical proofs. The author's methodology of formation modeling skills through GeoGebra cloud service based on a constructive approach is offered. The effectiveness of the proposed methodology was proved experimentally using the sign test.

Keywords

modeling skills, skills development methodology, cloud service, GeoGebra cloud service, constructive approach, modeling, interesting curves, professional training.

1. Introduction

Modern science operates with various methods, among which modeling is one of the most popular. It allows you to move away from the object's ideal representation and use its analogue, which retains the most important characteristics that allow you to talk about the object properties after certain changes or influences on it.

The development of computer technologies has contributed not only to the revival of the modeling method, but also led to the emergence of specialized environments, where it became possible to model various objects (processes) based on a constructive approach. At the same time, the widespread use of smartphones and tablets that have access to the Internet has led to the emergence of cloud services, which also allow you to model objects of different nature. However, methods of using cloud services to develop modeling skills are just beginning to

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be developed and implemented, and therefore are not well established and need experimental confirmation.

The problem of introducing cloud technologies in the educational process of secondary and higher education is in focus in the pedagogical literature.

Different aspects of educational using of cloud technologies and services are examined in the studies of national and foreign researchers. For example, A. Alkhansa , A. Shakeabubakor, E. Sundararajan and A. Hamdan [1] explore advantages of using cloud technologies in research, such as availability of various tools and applications for analyses and collecting data, for managing and organizing references, for communication with peers and experts, the absence of constraints, the access to cloud resources from anywhere any time that has an active Internet connection, etc. They offer the approach to applying of cloud services for enhancing the productivity of university research activities, increasing competitiveness and flexibility of educational institutions. S. Nabil [2] describes the main types of cloud services that reflect the possible uses of ICT outsourcing to create educational services. A. Smith, J. Bhogal and M. Sharma [3] core focus addresses Education as an area of cloud application with a cloud based e-learning system developed to demonstrate the capabilities and effectiveness of cloud technology.

O. Spirin and V. Ereemeev [4] substantiate the importance of the introduction of cloud technologies in education and consider the directions of their application in higher education, in particular the creation of a cloud-oriented educational and scientific environment in universities.

M. Shishkina and M. Popel [5] explore the problems of implementation of cloud technology tools and services in the educational process, describe the current state of development and use of cloud technology services in educational institutions and analyze the content of educational and scientific components of cloud-based educational environment.

Z. Seidametova, Je. Abljalimova, L. Medzhitova, S. Sejtvelieva and V. Temnenko [6] present the taxonomy of clouds, describe the methods of using cloud technologies in education, in particular, the approaches used in Google Apps for universities, Open Class, Piazza.

S. Litvinova [7] characterizes the cloud-based learning environment of the schools, the requirements for it, the purpose of creation, structural components, models of deployment and maintenance, describes its subjects and objects.

Scientists are considering cloud services for the formation of various specialists' skills in various fields of knowledge.

Currently, cloud versions of known environments with mathematical modeling capabilities are available, including Sage MathCloud, Maple, Maple Net, MATLAB, MATLAB web-server, WebMathematica, Calculation Laboratory, etc. [8]. We will also add CoCalc [9] and GeoGebra [10] cloud services to this list.

GeoGebra is a cloud service because it offers online data processing: viewing educational content created by others, online communication, development of own learning materials in the form of interactive applets [11], [12], [13].

We have substantiated the ways of using GeoGebra service [11]:

- for visualization of mathematical objects;
- for the organization of not only analytical, but also empirical search of the results at formulation of some characteristics of mathematical objects;

Table 1
Indicators of the formation of modeling skills

No	Indicators	Marks
1	Ability to take into account the analytical relationship between the elements	1
2	Ability to take into account the geometric relationship between the elements	1
3	Ability to use the Locus tool	1
4	Ability to use the Trace tool	1
5	Ability to demonstrate the change of the curve shape when changing the input data	1
6	Ability to construct a model visually correct (location, color, size and style of geometric objects)	1
7	Ability to add dynamic text to study numerical characteristics	1
8	Ability to write an algorithm for constructing a model	1
9	Ability to reproduce the steps of the algorithm to construct the model	1
10	Ability to interpret the result of a computer experiment	1

- for conducting a home computer experiment by students.

At the same time, the problem of the use of GeoGebra as a cloud service for modeling mathematical objects based on a constructive approach was not mentioned in the scientific literature. However, the recent spread of distance learning in universities shifts this problem in focus.

The purpose of the paper is to describe the method of forming the skills to model interesting curves through GeoGebra cloud service based on a constructive approach and to confirm its effectiveness.

2. Material and methods

To test the effectiveness of the methodology of forming modeling skills, a pedagogical experiment, which lasted 3 years (2018-2020) was conducted among pre-service mathematics and computer science teachers.

Makarenko Sumy State Pedagogical University, Borys Grinchenko Kyiv University were the experimental base. The total number of respondents was 86 people (pre-service mathematics and computer science teachers).

The author's methodology was implemented using the freely distributed GeoGebra cloud service (www.geogebra.org) in the special courses. The experimental module "Modeling of interesting curves" with a volume of 10 classroom hours (2 hours of lectures and 8 hours of laboratory classes) was introduced.

Two tests of educational achievements (at the beginning and after studying the module) were organized to confirm the effectiveness of the methodology. Students were offered a typical test problem for modeling a conic by its geometric definition (problems differed from each other in the input conditions, for example, the distance between the foci was different or one of the foci was in the specific point in a given coordinate system, etc.). The solution of the problem was evaluated by the following indicators (Table 1).

Since the results of the tests were dependent and each time provided for the accumulation of marks, the sign test was used. The number of respondents, whose total score decreased ("−"), did not change ("0") and increased ("+"), was fixed.

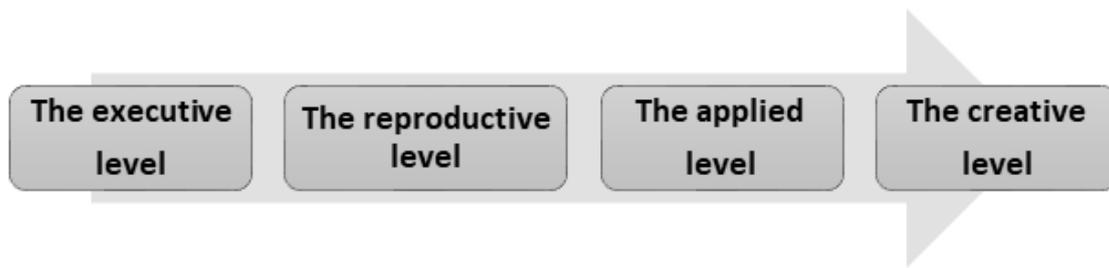


Figure 1: Constructive approach activity levels

In accordance with the purpose of the experiment and the sign test, the null hypothesis was formulated: the author's methodology does not contribute to the formation of modeling skills by means of GeoGebra cloud service. Then the alternative hypothesis was "the author's methodology contributes to the formation of such skills".

The constructed hypotheses define the one-sided sign test for checking the dependent samples. According to the decision-making rule [14] the null hypothesis of inefficiency / effectiveness of the author's methodology was accepted or rejected.

3. Constructive approach

The constructive approach is characterized by the fact that acquaintance with the properties of concepts begins with constructive activities for their "discovery" and assimilation with a gradual transition to definitions and logical proofs. This, in particular, simplifies the perception of the Geometry course, makes it more accessible, while raising the scientific level through the intensification of educatees' research activities.

The use of the constructive approach contributes to the fact that the activity is manifested in the gradual transition of actions for the construction of objects from the executive level (is characterized by external regulation), then to reproductive (is marked by internal regulation of actions in the construction of known structures), then to the applied level (using the method of construction), and, finally, the creative level (involves the construction of new objects) (Fig. 1) [15].

We consider constructive tasks to be one of the means of implementing a constructive approach. Many authors, in particular Yu. Kolyagin, A. Stolyar, L. Friedman, use the term "constructive tasks" in a sense identical to construction problems. However, the peculiarities of the constructive approach require the use for its implementation and other tasks, the solution of which requires the implementation of a particular constructive activity. Therefore, for example, O. Lysymova considers constructive tasks as tasks for construction, imaging, measurement, geometric design and structural-geometric modeling [16]. A. Dalinger considers constructive tasks as "problems in the process of solving which reveal the material conditions of geometric figures. Their purpose is to identify the essential features of ideas that are formed through the material conditions of their origin" [17].

Constructive tasks can be a base for establishing new properties of figures (concepts formation), for the consolidation of knowledge, for repetition, control, intensification of research

activities. "The peculiarity of constructive tasks is that they can be solved both logically and figuratively or visually effective" [16].

When solving constructive tasks, there is a productive activity that motivates to think independently (methods of constructing have to be developed independently), and not reproductive activity, which is often not an independent mental process, but is a repetition of known steps [18].

Mastering the system of knowledge and skills can take place on two levels: constructive and analytical. The main feature of the constructive level of mastery of the material is its visual and constructive awareness. This is manifested:

- in the ability to recognize objects that belong and do not belong to the content of this concept, give examples, demonstrate the existence of the studied figures by construction;
- in the knowledge of their most essential properties and the ability to apply known properties in solving problems.

The constructive approach provides "points of support" for Geometry study at a higher abstract level, which is characterized by the ability to formulate definitions of concepts, statements and prove already in formal language, rather than the language of geometric images.

4. Methodology of forming skills to model interesting curves based on a constructive approach in GeoGebra cloud service

Any methodology involves the presence of five elements (Fig. 2): purpose, content, forms, methods and means. Let us explain them in more detail.

The purpose of the author's methodology is to form the skill to model interesting curves using cloud services.

Interesting curves of the course of Analytical Geometry are the content on which the author's methodology is realized. Nowadays, their study in the classical course of Analytical Geometry is possible in three ways:

- analytical description and further study of curves – at first analytical (parametric, implicit, explicit) equations, usually of conics, are given and then they are investigated;
- study of curves as locus with a given property – at first the geometric definitions of curves are given, on the basis of which their analytical equations are written (as a rule, these are conics, conchoid of Nicomedes, limaçon of Pascal, strophoid, cissoid of Diocles, lemniscate of Bernoulli, Cassini oval);
- study of curves generated in the kinematic way (as the trajectory of a point), usually cycloidal curves, folium of Descartes, witch of Agnesi, logarithmic spiral.

Due to lack of time, there is only the first way of the study of interesting curves in the course of Analytical Geometry at universities. This limits the possibility to develop the skills to model mathematical objects.

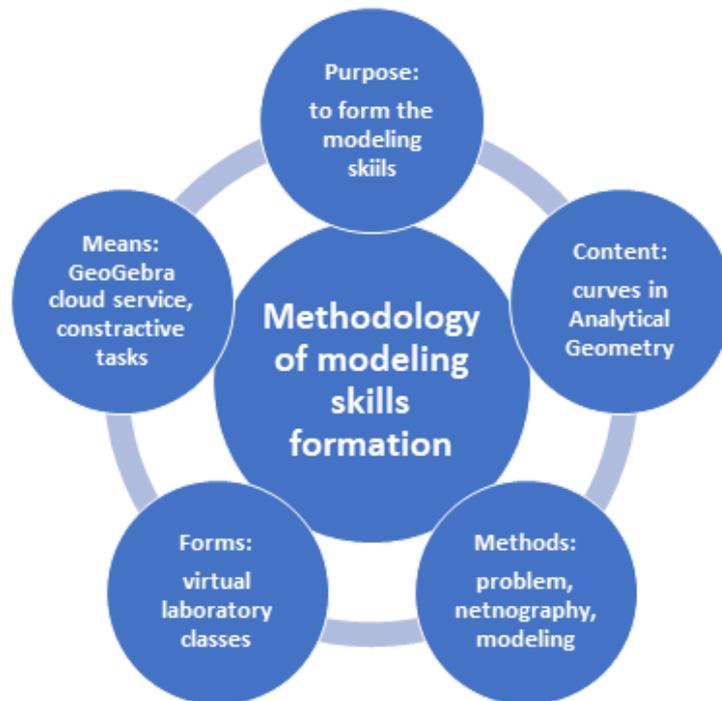


Figure 2: Components of the author's methodology

The form of realization of the author's methodology is virtual lecture-practical classes or a combination of visualized lectures and virtual laboratory classes.

Among the teaching methods problem method and netnography method are used.

GeoGebra cloud service, constructive tasks and instructional materials are learning means.

Elementary knowledge in Analytical Geometry is required to use the author's methodology.

In general, the methodology of forming the skills to model interesting curves is as follows: the teacher on a common online platform informs about constructive approaches to the construction of various curves, which were studied in ancient times (the netnography method). He briefly tells about how such constructions were done (compass and ruler, one compass, two compasses, one or two rulers, etc.), or gives an example of practically oriented problems that are solved using interesting curves. After that, the teacher demonstrates one of the described constructions in GeoGebra cloud service, and then asks students to write an algorithm for the above construction (the problem method). Then the teacher asks students according to the algorithm (provided to each student) to reproduce the construction and demonstrate the result, to analyze errors, and if not, to analyze possible limit cases. After that, students are offered the definition of curves generated in mechanical way together with the algorithms of their construction in the cloud service; students must model this type of curve according to the existing algorithm. Then together with students, the task of the following type is carried out: algorithms of curves construction are offered and after their construction, students need to characterize properties of the modeled curves, to give them definitions or kinematic characteristics. After completing this type of task, students are offered only the definition of the

curves or their kinematic characteristics, and they must model the curve themselves.

The methodology should be briefly described as follows.

Step 1 – the teacher offers an example of a curve model through the definition (the teacher step by step models the curve), and students must independently compile an algorithm for constructing the model.

Step 2 – the teacher offers a description-definition of the curve and provides a ready-made algorithm according to which students model the curve independently.

Step 3 – the teacher offers an algorithm for constructing a curve model, and students need to characterize the properties of the curve or give its definition based on the results.

Step 4 – students are offered definitions of curves that they have to model them.

We will briefly demonstrate the application of the methodology of forming the skills to model interesting curves based on a constructive approach in GeoGebra cloud service on the example of the hypocycloids study.

The teacher formulates the problem. The calculation of the action of the kneading machine consists of a description of the trajectory of the point belonging to the working body. The trajectory of the working body does not always fill the container with the substance, as a result, the mixing time increases, which leads to unnecessary energy consumption. Determine the optimal trajectory of the point of the mixer working body [19].

History of the curve discovery (netnography method). Hypocycloid (from Greek words $\upsilon\pi\omicron$ – "under, below" and $\kappa\upsilon\kappa\lambda\omicron\zeta$ – "circle") – is a curve "under the circle". The most famous hypocycloids are the astroid (from Greek "star-shaped") and the deltoid (Steiner curve). The name of the astroid (from "cusp" – exacerbation) was proposed by the Austrian astronomer Carl Ludwig Litrov (1811-1877). The shape of the deltoid resembles the capital Greek letter delta, its properties were studied by Leonard Euler in the XVIII century, and then by Jacob Steiner in 1856, after whom it was renamed the Steiner curve.

Definition of the curve. A hypocycloid is a curve generated by the trace of a fixed point on a small circle that rolls within a larger circle.

Next, the teacher demonstrates an example of a ready-made hypocycloid model (Fig. 3-4), and then offers to describe the algorithm for its construction to students.

Let the circle ω_2 roll inside the circle ω_1 . Let's construct a curve generated by a point on the circle ω_2 .

Algorithm for hypocycloid model constructing

1. Construct a circle ω_1 with center A passing through point B .
2. Construct the segment AB .
3. Construct an arbitrary point C on the radius AB .
4. Construct the circle ω_3 with center A passing through point C (dotted line).
5. Construct an arbitrary point D on the circle ω_3 .
6. Construct the circle ω_2 with center D , the radius of which is equal to the length of the segment CB .
7. Construct the ray AD .
8. Construct the point E – the intersection of the circle ω_2 with the ray AD .

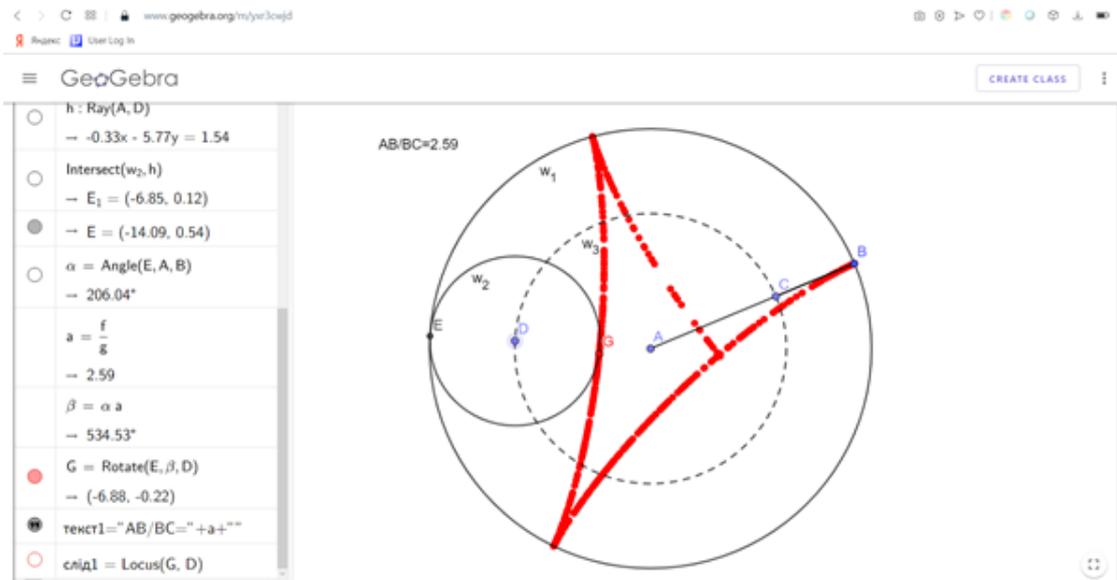


Figure 3: Construction of a hypocycloid using the Trace tool

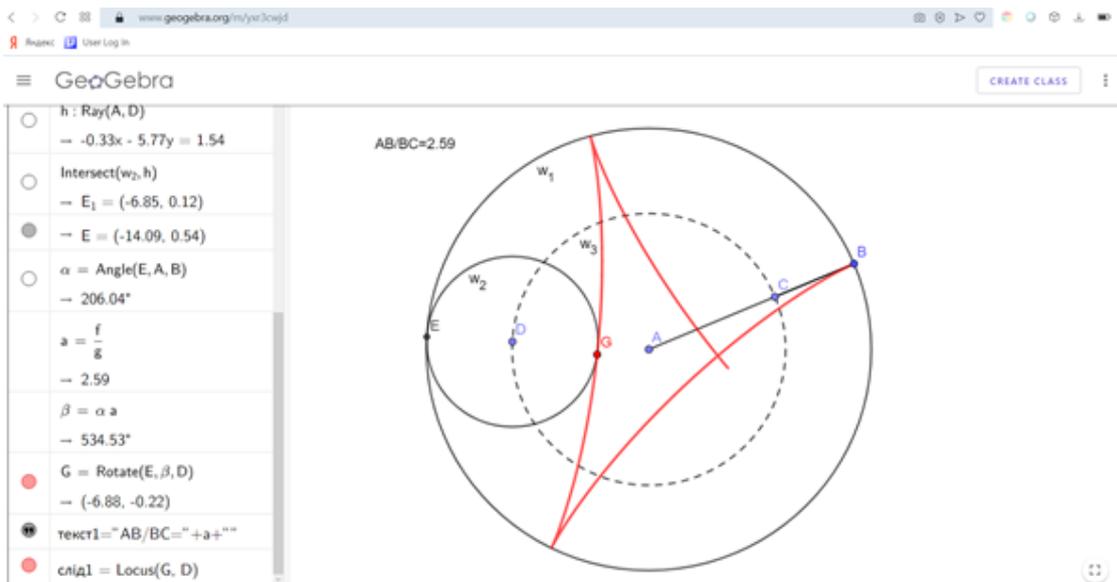


Figure 4: Construction of a hypocycloid using the Locus tool

9. Hide the ray AD .
10. Let the center D of the circle ω_2 coincides with the point C in the initial position. If we move the point D around the circle, it will rotate around the point A , and remains tangent to the circle ω_1 . Let the circle ω_2 rotate around it when moving the center to create the illusion that the circle ω_2 rolls on the circle ω_1 . Only one point of the circle ω_2 should

be investigated. Let this point coincides with the point B at the initial moment, and is the point of tangency of the circles ω_1 and ω_2 . If the circle ω_2 rolls on the arc BE , then the center D of the circle will return around the point A at the angle $\psi = BAE$. Simple calculations show that all points of the circle ω_2 will rotate around its center at the angle $\psi * \frac{AB}{CB}$.

11. Construct the point G , which is obtained from the point E by rotating around the point D by the angle $\psi * \frac{AB}{CB}$.
12. Hide the point E .
13. a) Make the point G leave a trace by rolling the point D on the circle ω_2 (Fig. 3).
b) Use the Locus tool: the point G is the "pencil point", the point D is the "driver point" (Fig. 4).

Next, the lecturer at the visualized lecture offers students to explore the analytical equation of the curve, which is described through the parameter.

Let R be the radius of a fixed circle, r be the radius of a rolling circle, $k = \frac{R}{r}$, then the parametric equation of a hypocycloid is

$$x = r * (k - 1) * (\cos\psi - \frac{\cos(k - 1) * \psi}{k - 1}), y = r * (k - 1) * (\sin\psi - \frac{\sin(k - 1) * \psi}{k - 1}).$$

The modulus of k determines the shape of the hypocycloid: $k = 2$ the hypocycloid is described by Tusi couple (diameter of a fixed circle), $k = 4$ by the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = R^{\frac{2}{3}}$.

Study of the curve and establishment of properties (problem method). When the hypocycloid is constructed, we pass to establishment of its properties at the level of practical research.

Note that the curve constructed in this way is open. Let us investigate the conditions under which the constructed hypocycloid will be a closed curve. By changing the radius of the circle ω_3 , thereby changing the ratio $k = \frac{R}{r}$ of the radii of fixed and rolling circles, we conclude that the curve will be closed when the ratio $k = \frac{R}{r} \in \mathbb{Z}$ (Fig. 5). In addition, in this case, the hypocycloid consists of k identical arcs, the ends of which lie on a fixed circle (Fig. 6).

If the radius r of the rolling circle is twice smaller than the radius R of the fixed circle, then the hypocycloid degenerates into a segment (Fig. 7). This statement is well known as Copernicus theorem.

After the lecture, students are first asked to construct curves models according to the given definition and algorithm, for example, to construct a cycloid, which is a curve generated by a point of a circle rolling on a straight line, reproducing all the steps of the algorithm. Not only instructions but also possible consultations are provided.

The next lesson tasks such as: "Establish the relationship between the number of arcs of the epicycloid, generated by a point of a circle rolling on the outside of another circle, and the ratio of the radii of these circles", are offered.

Then, at the next two lessons, tasks such as: "A point and a line, the distance between which is equal to a , are given. A line is drawn through an arbitrary point X of the line and the point. The points at a distance b from point X are marked. Find the locus of such points".

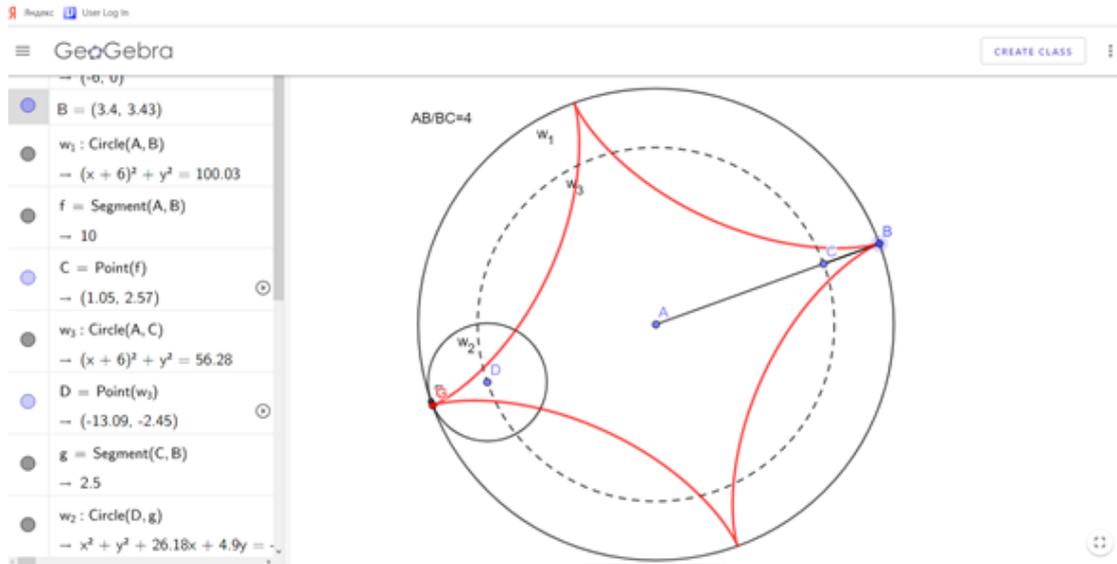


Figure 5: Closed hypocycloid, $k = \frac{R}{r} \in \mathbb{Z}$

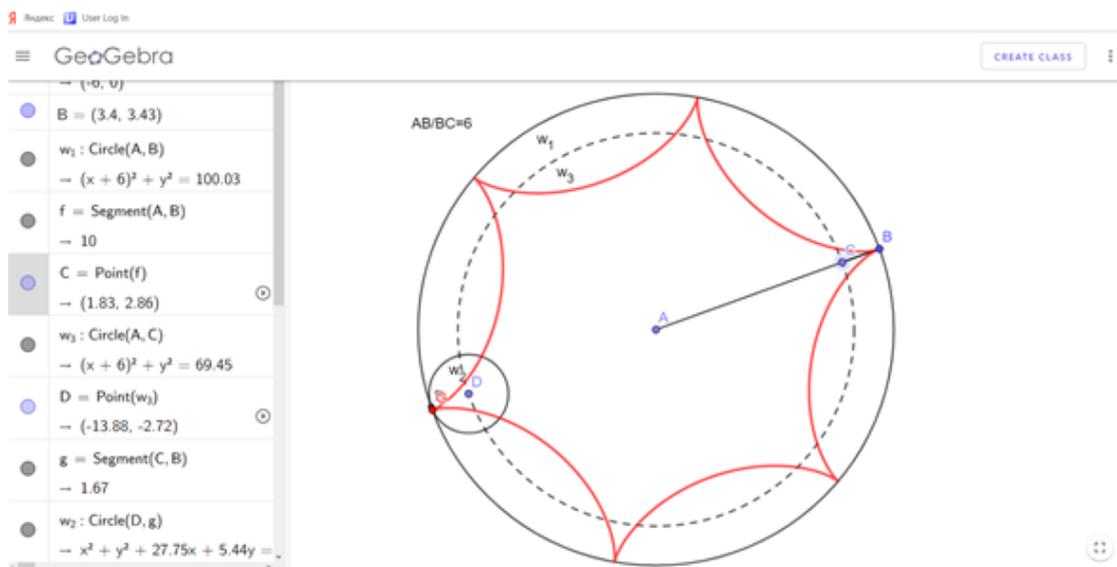


Figure 6: The hypocycloid consists of $\frac{R}{r} = 6$ of identical arcs

5. Statistical analysis of experiment results

This methodology of modeling skills has proven its effectiveness. Two tests of academic achievements were conducted at the beginning and after the study of the special course. Tests were performed according to the indicators shown in Table 1.

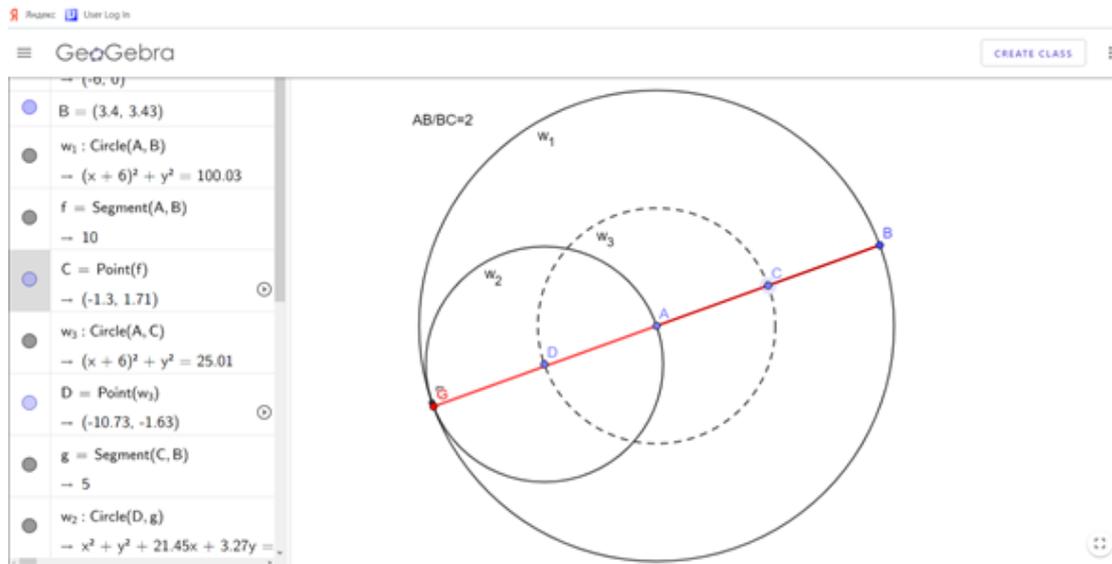


Figure 7: Degenerate hypocycloid, $\frac{R}{r} = 2$

The results of the controls are presented in Table 2.

These marks were used to determine the number of respondents whose total score decreased ("–"), did not change ("0") and increased ("+") (Table 3).

According to the decision-making rule [14] we have: the value of $T_{exp} = 46$ (this is the number of "+" signs in the sample), $n = 67$ (this is the number of respondents who have changed the results), the acceptance interval of the hypothesis H_0 is $[26, 41]$ at the significant level of 0.05.

Since T_{exp} is not included in the acceptance interval of the hypothesis H_0 , we reject the hypothesis H_0 and accept the alternative one with the conclusion that the author's methodology contributes to the formation of modeling skills. Since the value of T_{exp} is beyond to the segment on the right, it is necessary to conclude there is the positive dynamics of the number of such students who have developed the skills to model interesting curves using GeoGebra cloud service.

6. Conclusions

1. Author's methodology of forming modeling skills, which involves 4 steps: Step 1 – the teacher offers an example of a curve model through the definition (the teacher step by step constructs the curve), and students must independently compose the constructing algorithm; Step 2 – the teacher offers a description-definition of the curve and provides a ready-made algorithm by which students independently model the curve; Step 3 – the teacher offers an algorithm for constructing a curve model, and students based on the results need to characterize the properties of the curve or give its definition, Step 4 – students are offered to define curves that they have to model) confirmed its effectiveness by the sign test at the level of significance

Table 2
The results of students' tests

No	Test 1	Test 2	No	Test 1	Test 2	No	Test 1	Test 2
1	3	4	30	3	5	59	4	5
2	4	5	31	4	4	60	3	4
3	5	4	32	4	6	61	3	4
4	4	3	33	4	6	62	5	4
5	5	3	34	4	7	63	5	5
6	5	5	35	5	4	64	5	4
7	5	6	36	5	3	65	6	4
8	6	6	37	5	3	66	5	6
9	4	5	38	5	5	67	4	7
10	3	4	39	6	6	68	3	7
11	3	3	40	5	7	69	2	6
12	3	3	41	4	5	70	7	5
13	4	2	42	4	4	71	5	8
14	4	6	43	6	4	72	4	8
15	5	6	44	7	5	73	3	9
16	4	7	45	6	6	74	3	8
17	5	6	46	5	7	75	4	7
18	4	5	47	7	7	76	5	6
19	7	4	48	8	7	77	5	6
20	6	5	49	8	6	78	5	6
21	6	3	50	3	6	79	6	7
22	5	5	51	4	7	80	5	6
23	7	6	52	4	7	81	5	8
24	7	6	53	4	7	82	6	7
25	7	7	54	1	7	83	5	6
26	4	4	55	2	7	84	6	6
27	5	3	56	4	5	85	5	5
28	5	5	57	4	4	86	3	6
29	5	5	58	3	4	-	-	-

Table 3
Dynamics of marks according to the results of students' tests

Dynamics	Negative "-"	Without changes "0"	Positive "+"	Changes $n=\langle - \rangle + \langle + \rangle$
Members	21	19	46	25

of 0.05.

2. The use of GeoGebra cloud service is effective for the formation of students' modeling skills, pre-service mathematics and computer science teachers. At the same time, we consider it is also possible to use this methodology in the conditions of traditional training with GeoGebra installed on PC.

3. Construction of mathematical objects on the basis of a constructive approach contributes to the enrichment of the educatees' geometric experience, the formation of the world scientific

picture in terms of Geometry, because it provides the transformation of geometric information into visualized or analytical one (transcoding geometric symbols, graphics, spatial images, verbal descriptions) and obtaining new studied object properties on its basis.

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