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Abstract. The article considers the problem of the quality of mathematical school education in the context of pupils' ability to apply the acquired knowledge of mathematics to solve practical problems. It is established that this ability is a clear understanding and awareness of the role of mathematical knowledge in the modern world. It is manifested in the ability to explain natural and scientific phenomena, to draw sound conclusions, to understand the impact of science on human life. The results of a study in which 1849 pupils of Kyiv schools took part are presented. Their ability to apply mathematical knowledge to solve life problems has been established. Examples of mathematical problems developed for testing by specialists of the Ukrainian Center for Educational Quality Assessment are given. The typical pupils' mistakes, as well as the most successful solutions are analyzed. Pedagogical recommendations for improving the quality of mathematics education of pupils in the context of the reform of the "New Ukrainian School" are given.

1. Introduction

Mathematics can rightly be attributed to the key factors in the development of civilization. On the basis of mathematics, technical progress, computer science, computer science, etc. have been developed. Mathematical education in the system of general secondary education occupies one of the leading places, which is determined by its practical orientation, opportunities for human competence development. In addition, mathematical education is the key to personality formation, namely the development of higher processes of its thinking: judgments, inferences, comparisons, classifications, which ultimately affects the formation of intelligence. Attaching importance to this problem, in order to improve the quality of teaching and organization of education systems in the world, the International Program for Assessment of Student Achievement (PISA) includes assessment of pupils' knowledge in the field of natural sciences (*scientific literacy*), namely mathematics (*mathematical literacy*) [1]. In 2018, Ukraine took part in this survey for the first time. Actualizing the problem of mathematics education, the reform of the "New Ukrainian School" involves the development of pupils' critical and algorithmic thinking, developmental and problem-based learning, research skills, communication skills, project technology and etc. After all, it is this personal basis that enables the introduction

of modern teaching technologies aimed at forming pupils' ability to apply knowledge and skills, analyze, argue and communicate effectively in the process of solving and interpreting problems in different situations.

In Ukraine, in accordance with the "Basic Standard of General Secondary Education" [2], the requirements for compulsory learning outcomes are determined on the basis of the competence approach. Key competencies include:

- mathematical competence, which involves the ability to develop and apply mathematical knowledge and methods to solve a wide range of problems in everyday life; modeling of processes and situations with the use of mathematical apparatus; awareness of the role of mathematical knowledge and skills in personal and social life;
- competencies in the field of natural sciences, engineering and technology, involving the formation of a scientific worldview; ability and willingness to apply an appropriate set of scientific knowledge and methodologies to explain the world of nature; gaining experience in studying nature and formulating evidentiary conclusions based on the information obtained; understanding the changes caused by human activity; responsibility for the consequences of such activities.

The quality of innovative forms, methods and means of teaching mathematics is the subject of research of L. Sharoff [3], S. Vijayabarathi, K. Pramila, J. Sengamalaselvi [4]; in the context of the Covid-19 pandemic there are researches of S.A. Husain, N.A.E.M. Manan, V. Goergeshua [5], E.J. Sintema [6]. In the researches of Mailizar, A. Almanthari, S. Maulina, S. Bruce [7] the barriers to the implementation of distance learning of mathematics by means of digital technologies are identified, the main challenges facing the participants of the educational process (quality of education, interest in mathematics, etc.) are highlighted. It should be noted that the quality of mathematics education in the use of digital technologies is being studied by a number of scientists. Thus, M. Bano, D. Zowghi, M. Kearney, S. Schuck, P. Aubusson [8] presented an analysis of more than 60 studies of mobile learning in mathematics. M. Skryabin, J. Zhang, L. Liu, D. Zhang [9] have established the impact of digital technology on the quality of mathematics education at school. E. Makarova, B. Aeschlimann, W. Herzog [10] revealed the problem of gender stereotypes on the quality of study of natural sciences and mathematics. In a study by J.P.J. Van der Beek, S.H.G. Van der Ven, E.H. Kroesbergen, P.P.M. Leseman [11] the connection between the level of achievements in mathematics and the emotional state of a person is traced. Various aspects of the implementation of STEM-education are presented in studies by R. Christensen, G. Knezek, T. Tyler-Wood [12], S. Chachashvili-Bolotin, M. Milner-Bolotin, S. Lissitsa [13]. Important in the context of our study is the experience of M. Shield, S. Dole [14] on the development of textbooks in mathematics in the context of developing practice-oriented tasks. In recent years, our country has taken a number of initiatives aimed at improving school education in general and mathematics in particular, among them are:

- The Concept of implementation of state policy in the field of reforming general secondary education for the period up to 2029 "New Ukrainian School", 2016 [15];
- The Concept of development of science and mathematical education (STEM-education), 2019 [16];
- announcement of the 2020/2021 academic year as the Year of Mathematical Education in Ukraine [17].

Despite the above measures that are being implemented or have already been implemented in the educational process, the issue of the quality of mathematical training of school pupils remains extremely relevant. The outlined problem acquires special significance for Kyiv educational institutions, which usually serve as a reference point for the functioning and development of the education system for other regions of the country. However, according to preliminary results of

the evaluation of the quality of education, the capital's general secondary education institutions have certain problems in the context of science and mathematical training of pupils.

This encourages the scientific community to modernize and update mathematical training in school in accordance with the leading ideas of the concept of "New Ukrainian School", which among the key competencies necessary for successful self-realization in society, defines mathematical competence.

2. The aim of research

The purpose of the study is to analyze the results of pupils' diagnostics on the ability to apply mathematical acquired knowledge to solve practical problems, to develop recommendations for improving the quality of mathematics education at school.

3. Methodology

The respondents were 15-year-old pupils from schools in Kyiv. In most countries, it is that age when pupils graduate from general school, faced with a choice of profession and future life path in general. The mathematical test was prepared by the staff of the Ukrainian Center for Educational Quality Assessment. The test was prepared in accordance with the requirements of the International Program for Assessment of Student (Pupil) Achievement PISA [1]. The test consisted of 10 tasks, which took 20 minutes. The semantic block of the test consisted of the following topics: "Numbers and expressions", "Equations and inequalities", "Functions", "Triangles, quadrilaterals, circles", "Vectors and coordinates".

Pupils were offered tasks of various forms:

- task with the choice of one correct answer. Each task had a basis and four possible answers, of which only one was correct. The task was considered completed if the test participant chose and correctly marked the answer;
- the matching task had a base and two columns of information marked with numbers (left) and letters (right). Execution of the task involved establishing a correspondence (formation of "logical pairs") between the information marked with numbers and letters. The task was considered completed if the test participant made marks at the intersection of rows (numbers from 1 to 4) and columns (letters from A to D) in the table;
- open-ended task with a short answer.

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4. Results and discussion

The following provisions were the guideline for the implementation of the study:

- The quality of mathematical training of the younger generation is an indicator of the readiness of society for socio-economic development, mobility of the individual in the development and implementation of modern technology, new technologies.
- Mathematical education is an important component of general education. The place of mathematics in the school system is determined by its role in the formation of educational, social, cultural and life competencies, values of civil society, pupils' personal development with a focus on continuing education, in the formation of creativity and critical thinking, creativity.
- Mathematics is one of the basic subjects of general secondary education, which provides a successful study of other disciplines, especially the subjects of the natural science cycle [18].

The study did not aim to find out how well pupils mastered the content of the school program. Their ability to use the knowledge, skills and abilities acquired at school to overcome real life difficulties and challenges was assessed. This ability is a clear understanding of the role of natural and mathematical knowledge in the modern world, the ability to explain natural and scientific phenomena, draw sound conclusions about them, understand the impact of science and technology to improve the material, intellectual and cultural environment. In this aspect, the study echoes the idea of the International Student Assessment Program (PISA), the results of which in Ukraine in 2018 confirmed the need to strengthen and modernize the mathematics education of pupils in general secondary education. The study, which took place on September 29, 2021, involved 1,849 10th graders, representing 292 Kyiv schools. They were asked to complete tasks for the 9th grade of four levels of difficulty (beginner, intermediate, sufficient and high). In addition to identifying pupils' ability to apply knowledge to solve practice-oriented problems, it was also important to assess their "residual knowledge" as part of the learning material that remains in the memory after learning the discipline and is sufficient for further study. The following results of solving mathematics problems are obtained:

- pupils gave the correct answer for 49.5% of tasks;
- pupils gave the wrong answer for 30.4% of tasks;
- pupils did not provide answers for 20.1% of tasks.

According to the results of the research, on average, each pupil correctly solved half of the tasks of a practice-oriented nature, which can be attributed to the average level of ability. Interestingly, a survey of 254 mathematics teachers of Kyiv schools found that teachers also assessed pupils' ability to apply knowledge in practice as average (see figure 1). Note that the teacher survey was conducted to identify real problems of natural and mathematical education of pupils. As we

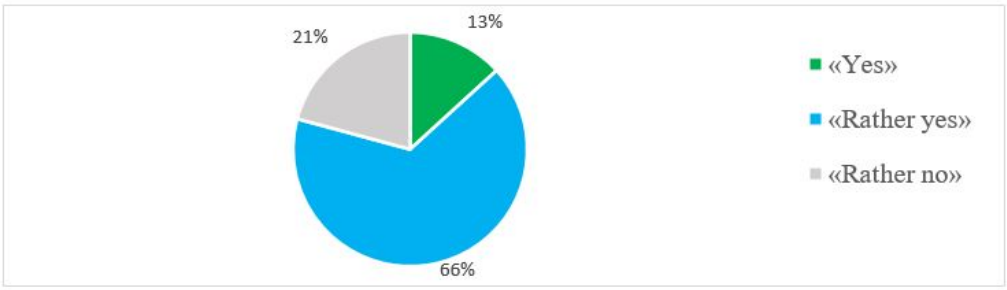


Figure 1. Distribution of teachers' answers to questions: "In your opinion, are pupils able to apply theoretical knowledge in practice to overcome various life challenges and problems?",%.

can see, only 13.2% of teachers answered yes to this question, the majority hesitated, choosing the alternative "Rather yes" (66.0%). This outlines the importance of implementing the applied orientation of academic disciplines, the development of skills to "see" mathematics in the real world, to apply the knowledge gained in school to solve everyday life needs. In this aspect, the recommendations of the International Comparative Studies (TIMSS, PISA, etc.) can be useful. The study found that 55.9% of teachers partially take them into account in their work. 34.5% of people answered in the affirmative to this question. At the same time, only 20.7% of pupils reported their experience in performing practice-oriented tasks, which indicates the need to solve the problem of improving pupils' ability to apply theoretical knowledge in practice to overcome various life challenges and problems. It is also interesting that according to teachers, the quality of mathematics education correlates with the introduction of interactive learning technologies, the use of digital tools and services, the promotion of group work to increase pupils' interest in

learning mathematics. Let's analyze the success of mathematical tasks. The study found that the best pupils solved problems on the following topics: "Numbers and expressions. Numerical sets", "Functions. Formula", "Coordinates and vectors. Coordinates of the point". The range of correct answers ranged from 84.2% to 90.4%. With the help of these tasks with practical content tested knowledge of numerical sets, the ability to correlate the desired number with the numerical set to which it belongs, to express the formula of the relationship between two variables, to determine the coordinates of a point. Let's analyze some test tasks (figure 2, 3).

Task 1. What is the number of participants in a video conference?

A $\sqrt{160}$
B $\frac{200}{3}$
C 18
D -15

Figure 2. Test task 1.

Task 1 statistics show that 8.4% of respondents did not understand the conditions of the task and believed that the number of participants in the video conference could be non-integer or negative. To determine the correct answer, it was necessary to find out that the number 160 is not a square of a natural number and that the number 200 is not divisible by 3.

Task 2. 51 jars with sauces and jams were packed in gift sets, each of which contains either 3 jars with sauces (x sets) or 4 jars with jams (in sets).

Specify the correct equality.

A $3y + 4x = 51$
B $3x + 4y = 51$
C $\frac{x}{3} + \frac{y}{3} = 51$
D $12xy = 51$

Figure 3. Test task 3.

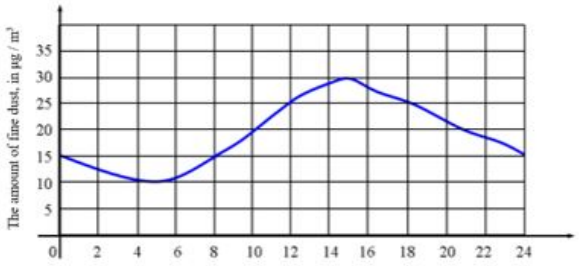
Building a mathematical model (task 2) is an important mathematical competence needed to solve text problems. One tenth of the participants did not manage to compose a letter expression with variables, which determines the number of cans of food. An important element was the drawing, which visualized the condition of the task and facilitated its implementation. Quite successfully students coped with the tasks on the topic: "Numbers and expressions. Text tasks" and "Functions. Function schedule". The range of correct answers is from 63.2% to 75.9% (figure 4, 5).

With the help of task 8 the formation of such subject competence as the ability to solve a text problem in an arithmetic way was tested. The task was to understand that an inflatable boat moves on a river at the speed of its flow and to determine the number of hours tourists spend rafting on the river, using the formula for the length of the route from rafting time and boat speed. Note that 30% of participants did not cope with this task.

Task 8. Tourists rafting down the Tisza River on inflatable boats without engines, using oars only to avoid obstacles. The length of the rafting route is 6 km. The speed of the Tisza along the entire route is 1.5 km / h. How many hours will tourists spend rafting on the river?

Figure 4. Task 8.

Task 9. Fine dust in the air is its main pollutant. The figure shows the change in the amount of this dust (in μg / m^3) in a certain area of the city during the day. Match the question (1-4) to the answer (A to D).



Question:

1. What amount (μg / m^3) of fine dust was recorded at 8.00?
2. At what time was the fine dust the least?
3. For how many hours was observed an increase in the amount of fine dust?
4. For how many hours did the amount of fine dust exceed the permissible average daily level of 25 μg / m^3?

Answers:

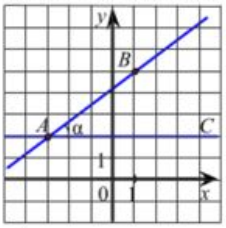
A 5
B 6
C 10
D 15
E 20

Figure 5. Task 9.

With the help of task 9 the formation of such subject competencies as the ability to determine was checked: the graph of the value of the function on a known argument, the value of the argument on a known function, the intervals of growth of the function. The task was to establish the relationship between time and the amount of fine dust, which is one of the factors of air pollution. To do this, it was necessary to determine the value of the function (the amount of dust recorded at this time) from the known value of the function argument (time), find the range of argument values for which the function values exceeded the specified number, specify the argument corresponding to the smallest function value. There is significantly lower performance of tasks on the following topics: "Coordinates and vectors. Coordinates of the vector", "Coordinates and vectors. The distance between two points", "Triangles, quadrilaterals, circle". The range of correct answers is from 30.2% to 46.2%. These tasks tested the ability to determine the coordinates of the vector; determine the distance between two points in the Cartesian coordinate system on the plane; determine the length of the segment by its parts, the length of the semicircular arc, the sides of the rectangle. The tasks presented practically indicative problems for determining the coordinates of the vector and its module as the distance between the school and the museum; on the correspondence between a certain value and its

value on the example of a window frame. Less than half of the participants determined the coordinates of the vector in the figure showing its beginning and end, so they did not use the required formula. The length of the vector could be determined not only by the appropriate formula, but also by using Pythagoras' theorem to find the unknown hypotenuse by known legs (see figure 6). According to the results of the study, the most difficult tasks were: "Equations

Task 5. In a rectangular Cartesian coordinate system on the plane marked the location of the school (point A) and the museum (point B). The rectilinear road AB passes directly near the school and the museum, the rectilinear road AC passes near the school and is parallel to the x-axis (see figure).



Determine the coordinates of the vector $\overline{AB}$.

Figure 6. Test task 5.

and inequalities. Systems of equations", "Numbers and expressions. Finding the percentage of a number", "Triangles, quadrilaterals, circle. Sine, cosine, tangent of the acute angle of a right triangle. The relationship between the sides and angles of a right triangle", "Numbers and expressions. Text tasks". According to the results of the study, the range of correct answers is from 11.5% to 14.0%. With the help of these tasks the ability to compose a system of equations and find its solution was tested; find the percentage of a number; determine the angle between the lines on the plane and find the trigonometric function of this angle from a right triangle; build a mathematical model of a text problem and solve it algebraically. Only 11% of participants were able to build a mathematical model of the problem (Task 3), which is a system of linear equations, and solve it. Slightly more (14%) participants made a mathematical model for the problem in the form of a linear equation and found its root. Determining the cosine of an acute angle in a right triangle is a basic skill for planimetry, but it is difficult for most (90%) participants. The scalar product formula could also be used to determine it, but this topic is even more difficult. We present some tasks from the test (figure 7, 8).

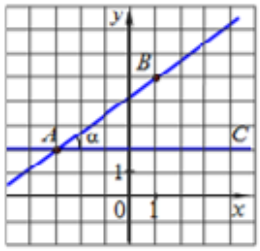
Task 3. 51 jars with sauces and jams were packed in gift sets, each of which contains either 3 jars with sauces (x sets) or 4 jars with jams (in sets). A total of 15 sets were completed.

1. Determine the number of jam sets.
2. What percentage is the number of jam sets out of the total number of sets?

Figure 7. Test task 3.

In the study, we also found the results of the test in mathematics, taking into account the attitude of pupils to the problem of applying the acquired knowledge to solve practical problems. In most cases, those pupils who feel the importance of natural sciences and mathematics (answer "Yes" to the question), received higher scores on the test results. Such pupils received an average of 9.8 points, others received 9.3 points out of 20 possible. Given that for the second year in a row the learning process takes place in a mixed format, the study was important to identify the impact of distance education as a factor in the quality of mathematics education of pupils.

Task 7. In a rectangular Cartesian coordinate system on the plane marked the location of the school (point A) and the museum (point B). The rectilinear road AB passes directly near the school and the museum, the rectilinear road AC passes near the school and is parallel to the x-axis (see figure).



Determine the cosine of the angle α between the roads AB and AC.

Figure 8. Test task 7.

The results of the survey of teachers on the scope of distance learning show that pupils of those schools where, according to teachers, all pupils are covered by distance learning, received the highest scores in testing (average scores 9.6 and 8.4, respectively). As a result of the research, it was also interesting to investigate is there a relationship between the forms of test tasks and the success of their implementation? Pupils were presented with a task with the choice of one correct answer, a task to establish compliance, an open-ended task with a short answer: a structured task and an unstructured task. It was found that pupils performed the following tasks best:

1. to choose one correct answer of the initial and intermediate levels of difficulty (68.2% of correct answers were provided);

2. to establish compliance with high to advanced levels of difficulty (provided 52.9% of correct answers).

Open-ended tasks were much more difficult for pupils: 27.2% of correct answers (structured task), 25.4% of correct answers (unstructured task) of various types of complexity other than the initial one. Thus, according to the results of the study, improving the mathematical training of pupils in Kyiv is a complex and multi-vector process that can be implemented in synergy of its various actors. Here are methodological recommendations for improving the quality of mathematics education of pupils in the context of the reform of the "New Ukrainian School":

1. Increasing the attention of teachers to the study of those learning topics that caused the greatest difficulties for pupils in the testing process: "Equations and inequalities. Systems of equations", "Numbers and expressions. Finding the percentage of a number", "Triangles, quadrilaterals, circle. Sine, cosine, tangent of the acute angle of a right triangle. The relationship between the sides and angles of a right triangle", "Numbers and expressions. Text tasks".

2. In accordance with the requirements of the State Standard of Basic Secondary Education, teachers of natural sciences and mathematics of 5-9 classes focus on revealing the competence potential of mathematics and natural sciences, in particular, focus on real practical, life problems that are relevant to pupils and motivate them to learn.

3. When assessing academic achievement, the attention have been payed to the ability of pupils to apply the acquired knowledge of natural and mathematical disciplines to solve practical problems, actively use tasks of interdisciplinary practice-oriented nature.

4. Introduce interactive learning technologies, strengthen team learning activities; use of digital

tools for demonstrations, simulations of experiments, to promote group work, electives to increase the level of interest of pupils in the study of natural sciences and mathematics.

5. Conclusions and prospects for further research

1. The study found that pupils demonstrated an average level of ability to perform mathematical tasks of a practice-oriented nature. On average, 49.5% of tasks were completed, and the range of correct answers for different tasks ranged from 11.5% to 90.4%.

2. The most successful pupils solved the problems with the topics: "Numbers and expressions. Numerical sets", "Functions. Formula", "Coordinates and vectors. Coordinates of the point". The range of correct answers ranged from 84.2% to 90.4%. In solving these problems, the formation of the following competencies to solve problems of practice-oriented nature was tested: knowledge of numerical sets, the ability to correlate the required number with the numerical set to which it belongs; the ability to formulate the relationship between two variables; ability to determine the coordinates of a point.

3. Pupils made the most mistakes when solving problems on the following topics: "Equations and inequalities. Systems of equations", "Numbers and expressions. Finding the percentage of a number", "Triangles, quadrilaterals, circle. Sine, cosine, tangent of the acute angle of a right triangle. The relationship between the sides and angles of a right triangle", "Numbers and expressions. Text tasks". The range of correct answers ranged from 11.5% to 14.0%. With the help of these tasks the ability to compose a system of equations and find its solution was tested; find the percentage of a number; determine the angle between the lines on the plane and find the trigonometric function of this angle from a right triangle; build a mathematical model of a text problem and solve it arithmetically.

4. Recommendations for improving the quality of mathematics education at school in the context of the implementation of the reform "New Ukrainian School" (increasing the attention of teachers to the study of educational topics that caused the greatest difficulties for pupils in the testing process; introduction of interactive learning technologies, use of digital tools for demonstrations, popularization of group work to increase pupils' interest in learning mathematical disciplines; active use in the learning process of interdisciplinary practice-oriented tasks; ensuring the development of the internal quality assurance system of education) are highlighted. Methodological support for their implementation requires additional study.

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