



THE TEACHER'S ROLE IN THE FORMATION OF MATHEMATICAL COMPETENCE THROUGH SYMMEDIAN RESEARCH

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ABSTRACT

Purpose: The research is of significant importance as it focuses on evaluating instructional methods used by mathematics teachers to promote intellectual growth and develop critical thinking skills. The study aims to analyse the main teaching strategies that ensure students' desire for independent learning.

Design/Methodology/Approach: The study adopts a mixed-methods approach, integrating qualitative and quantitative research paradigms to explore how modern approaches and practices can enhance professional mathematical competence among teachers. The exploration focuses specifically on the application of symmedian research in mathematical instruction. The research is based on solving problems in geometry, in particular, on studying the properties of the symmedian of a triangle, which contributes to the development of spatial perception and analytical thinking.

Findings: The integration of various methods and technologies is crucial in developing mathematical competence, preparing students for solving complex problems in a globalised world. The article's recommendations for using interactive tools, collaborative platforms, and adaptive software to improve math teaching, as well as its directions for optimising digital technologies and innovative methods in teaching mathematics, are of significant value.

Research Limitation/Implications: The article underscores the teacher's crucial role as a mediator who adapts traditional teaching methods to meet the demands of the modern educational environment. It highlights the practical implications of preparing students to thrive in a rapidly evolving world, making the research findings directly applicable to the professional practice of educators.

Social Implication: This paper highlights the importance of teachers acting as mediators who can adjust traditional teaching methods to meet the needs of the contemporary educational environment.

Practical Implication: This study underscores teachers' need to possess practical teaching skills, which are pivotal for ensuring students successfully assimilate and comprehend educational material.

Originality/ Value: By integrating symmedian concepts into the curriculum, this study offers new insights into how teachers can more effectively develop mathematical competence, thereby contributing a unique angle to the existing academic discourse on mathematical education.

Keywords: *Antiparallels. Cheva's theorem. isogonal lines. lemoine. symmedian*



INTRODUCTION

The formation of mathematical competence is an essential element of the development of the intellectual potential of students, which affects all areas of their lives and education. According to Willemsen (2023), mathematics includes many structural elements, the most effective of which are algebra and geometry. Each provides students with tools to develop critical thinking and problem-solving skills. Geometry uniquely shapes spatial representation, which is critically essential in mathematics and many other fields, from engineering to art. The study of geometric shapes, theorems and proofs supports logical thinking and helps students visualise and solve complex problems, developing the ability to go beyond standard algorithmic thinking. The development of geometric thinking is vital for solving abstract mathematical problems. A teacher's mathematical competence is aimed at the ability to interpret the physical world around, which is the basis for many applied sciences and technologies.

Approaches to teaching mathematics and the variety of possible solutions to specific problems are critically important in stimulating teachers' and students' independence and competence. Viseu, (2022) paid attention to an individual approach, taking into account the importance of adapting lessons to the personal needs of each student. Striving for diversity in solving problems allows teachers to show various ways to solve problems. The appropriate method contributes to the reliability of understanding mathematical concepts and develops students' flexibility in thinking. Teachers constantly expand their approaches and strategies to encourage independent learning, asking questions, and finding answers.

The spread of digital infrastructure and innovative methods is crucial for modern mathematics education. According to Sukestiyarno (2023), using digital tools and resources in interactive platforms, virtual laboratories, and educational programs allows students to gain experience working with mathematical concepts in a dynamic format. Digital technologies open opportunities to demonstrate mathematical principles, providing practical and motivating learning. Integrating digital technologies into mathematics education stimulates students to develop digital literacy, which is necessary for successful adaptation to the technological world. The modernisation of education provides teachers access to various resources for assessment and individualisation of the learning process. The purpose of the study is to investigate the mathematical competence of teachers. The primary objective of this research is to identify the most effective strategies for supporting independent mathematical study and to examine the role of digital technologies in contemporary mathematics education. The research focuses on identifying and advancing pedagogical innovations that strengthen students' autonomy and contribute to their deep mathematical understanding.

LITERATURE REVIEW

Research in mathematics education indicates that the quality of teaching mathematics significantly impacts students' cognitive and motivational development. Facciaroni et al. (2023) showed that different trajectories of students' perception of a teacher's competence in mathematics are closely related to their involvement in learning and achievements in this field. According to Kontsevich et al. (2023), those students who independently completed tasks from mathematics teachers were more likely to succeed in higher education.



Assar and Grimes, (2023) emphasise the importance of the role of the teacher as a researcher in reforming mathematics education. Prokopenko et al. (2020) analyse the optimisation of business processes within the digital economy, underscoring the critical role of technological advancements in enhancing operational efficiency. Kasych et al. (2021) delve into the pivotal influence of intellectual capital on the innovative capacities of companies, highlighting the necessity for organisations to harness knowledge and creativity for sustained growth. Bielialov, (2022) explores the intricacies of risk management strategies for startups developing innovative products, emphasising the importance of proactive risk assessment in the entrepreneurial landscape.

Megits et al. (2022) conducted the “Five-Helix” model as a comprehensive framework for business development in the Industry 4.0 era, advocating for a collaborative approach that integrates various stakeholders to drive innovation and progress. Jia et al. (2024) note that teachers should be considered vital stakeholders, strengthening the link between theory and practice. Fino et al. (2023) examined modern approaches to teaching mathematics, which promote the independence and competence of students, which are becoming more and more popular. Han et al. (2023) argued that using digital technologies in education is essential to independent education. Boonstra et al. (2023) conducted a study to determine the professional development of teachers, which shows that the interactive components of classes are closely related to improving their mathematical competence. Yaniawati et al. (2023) determined that the value-based approach in the education of future mathematics teachers emphasises the importance of forming semantic competence.

According to Pamfilos, (2021), specific challenges may arise when implementing interactive approaches in the study of geometry. According to Müllerand Montúfar, (2023), changes in the teacher’s role under digital modernisation require new qualities from teachers, such as encouraging independent learning among students and mastering a mathematical approach to teaching. According to the results of the study of Zlepalo and Jurkin, (2018), there is not always a direct relationship between the observed competence of the teacher and the mathematical achievements of students.

Bertagna et al. (2023) claim that the effectiveness of teaching mathematics is closely related to using the latest methods and approaches that stimulate independent learning. According to Viseu et al. (2022), mathematical competence development in geometry can manifest in the concepts of Cheva’s theorem and the Lemoine point. The work of Magdalena-Benedicto et al. (2023) emphasises a deep understanding of geometric concepts that contribute to students' logical thinking and analytical abilities. De Vink et al. (2023) indicate the need to integrate interactive technologies into the educational process for better assimilation of abstract mathematical theories by students. According to the analysis of Bergshoeff et al. (2023), using digital resources significantly improves the ability of students to learn independently in complex geometric constructions. Nogueira et al. (2023) stated that improving teachers' digital literacy is critical for effectively implementing tools in the educational process.



The work of Quinn et al. (2023) analyses that modern pedagogical approaches focus on cognitive methods because they improve students' understanding of mathematical concepts. Cheng and Wu (2024) emphasise the importance of involving students in discovery and exploration instead of traditional memorisation. Sukestiyarno et al. (2023) suggested that developing effective teaching strategies that include a geometric component in imagination development will be the basis for training the next generation of mathematicians and engineers. Therefore, integrating innovative technologies and activating independent learning through a deep understanding of critical mathematical concepts are crucial for forming a strong mathematical foundation in students.

MATERIALS AND METHODS

This study employs a quantitative and qualitative methodology to analyse the efficacy of modern approaches in enhancing professional mathematical competence. The research centres on problem-solving within geometry, particularly through exploring the symmedian properties of a triangle to foster spatial perception and analytical thinking.

The target population includes secondary school students currently enrolled in advanced mathematics courses. A practical sample of students, focusing on those actively engaged in advanced mathematics courses, was selected from classroom practice sessions to participate in the study.

Data were collected through direct classroom observations, student surveys, and assessments focused on specific geometry problems, including studying Cheva's theorem and applying isogonal lines. These tools were designed to measure the depth of mathematical understanding and students' creative imagination development.

Quantitative data analysis involved statistical evaluation of problem-solving effectiveness and student engagement with graphical learning methods. Qualitative analysis included content analysis of student responses and a critical review of educational resources. The findings demonstrate the positive impact of innovative pedagogical approaches on students' ability to visualise mathematical concepts and apply theoretical knowledge practically. Digital technologies and interactive platforms were also assessed, showing significant benefits in simplifying traditional mathematical methods and enhancing student learning experiences.

The study confirms that integrating advanced geometric problem-solving into the curriculum significantly enhances students' mathematical competencies. This research not only supports the use of symmedian properties in teaching but also highlights the role of digital tools in modernising educational practices and fostering independent learning and critical thinking among students.

FINDINGS AND DISCUSSION

Constant improvement of a mathematics teacher's skills requires an understanding of the subject's theoretical foundations and the ability to use knowledge in the educational process and transfer it to students. A mathematics teacher creates a stable, meaningful learning



environment that helps students learn the material. To achieve such competence, the teacher conducts a constant dialogue between his knowledge and teaching methods, looking for ways to make mathematics accessible and understandable for every student.

Increasing competence occurs through continuous self-improvement through professional courses, independent study and collegial discussion. It effectively improves knowledge of algebra, geometry, and trigonometry and supports modern educational trends in differentiated learning and integrated STEM learning. Possessing various pedagogical methods and strategies allows the teacher to adapt learning to students' individual needs, forming lessons that respond to different learning styles and ensuring the involvement of the entire class. Transferring one's competence to students is an art that requires the teacher's knowledge and ability to connect with students, develop their interests, and cultivate their ability to learn independently. The quality of teaching in the classroom is directly correlated with the level of teacher training, his ability to update and apply his knowledge in the educational process, and his ability to inspire students to self-education.

The teacher's role in traditional teaching is to explain theorems and problem-solving methods through direct instruction and demonstration. Teachers usually focus on imparting knowledge about properties of shapes and theorems that can be applied, for example, Cheva's theorem or Lemoine's theorem. Standardised methods of deriving answers are widely used. Consider an example of using a figure to illustrate how to derive the required properties with a dot on the side. Given a triangle ABC (Figure 1). $AC = b$, $AB = c$. AD - symmedian. Find: $BD:CD$.

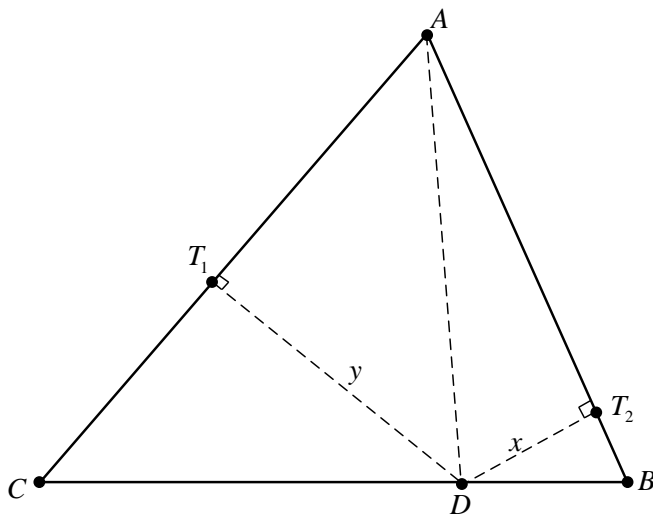


Figure 1. A triangle with properties for a symmedian
 Source: developed by the author

The solution.

$$\frac{x}{y} = \frac{c}{b}, \text{ Besides: } \begin{cases} DB = \frac{x}{\sin B} \\ CD = \frac{y}{\sin C} \end{cases}, \Rightarrow \frac{DB}{CD} = \frac{x}{y} * \frac{\sin C}{\sin B} = \frac{c}{b} * \frac{c}{b} = \frac{c^2}{b^2}.$$

Main property: $\frac{DB}{CD} = \frac{c^2}{b^2}.$



Answer: c^2/b^2 .

By moving from traditional teaching to an approach that encourages independent learning, the teacher becomes a mentor and facilitator who creates the conditions for students to explore and come up with answers independently. Instead of directly explaining the solution to the problem, the teacher can offer students a project or a series of investigations that lead to their conclusions about the properties of triangles. Students can use dynamic geometry software to manipulate the elements of a problem, visualising how changing the location of points affects other triangle elements and the corresponding relationships, stimulating a deeper understanding of geometry.

Mathematical competence development focuses on correct problem-solving, logical thinking, and the ability to analyse, think critically, and apply knowledge in new situations. When considering a triangle problem concerning a symmedian, the teacher can allow students to develop their methods of proof or alternative solution paths that involve finding relationships between the triangle elements. The appropriate approach allows students to become active participants in the educational process, contributing to the formation of mathematical skills and abilities.

In a traditional classroom environment, a teacher uses a symmedian to solve standard geometry problems. The teacher uses such problems to show how to work with geometric shapes, determine central and inscribed angles, and apply theorems to solve them. For example, you can set the following task - to find the symmedian of a right triangle, which comes from the vertex of a right angle, if its legs are equal a and b .

Given a triangle ABC ($\angle C = 90^\circ$). $AC = b$, $BC = a$. CO is the median. $CH \perp AB$, CH – symmedian (Figure 2). Find: CH .

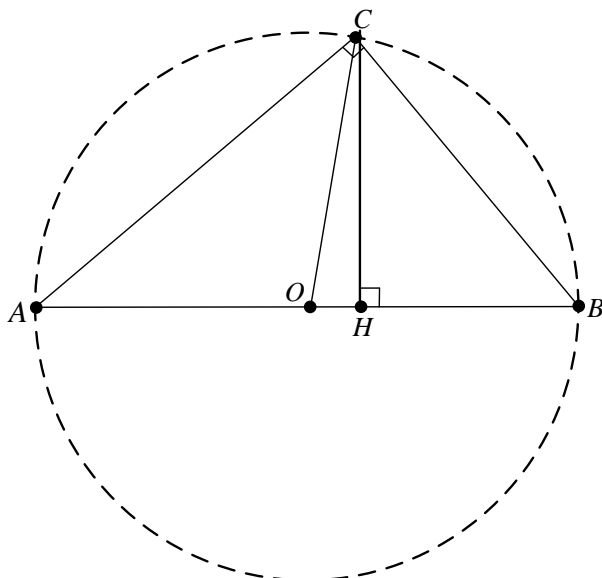


Figure 2. A triangle in a circle with a symmedian
Source: developed by the author



To develop mathematical competence, it is necessary to teach students to think mathematically, determine problem-solving strategies, and come to conclusions independently. Circle and triangle problems are used in architecture and other design sciences. Competence includes identifying and solving complex problems, working with abstract concepts, and understanding deep mathematical principles corresponding to those presented in Figure 2.

1. $AO = OB \Rightarrow CO$ is the median.

Triangle AOC - isosceles: $\angle OAC = \angle ACO = a$.

Triangle CHB - rectangular: $\angle OBC = 90^\circ - a$, $\angle BCH = a$.

So, $\angle ACO = \angle BCH = a$ or CO is the median, and CH is the symmedian of the triangle ABC , which are drawn from the top C .

$$2. CH = \frac{CA \cdot CB}{AB} = \frac{ab}{\sqrt{a^2 + b^2}}.$$

Answer: $\frac{ab}{\sqrt{a^2 + b^2}}$.

Applying Cheva's theorem and analysing the properties of symmedians open up opportunities for students to conduct independent research. The teacher, acting as a mentor, can provide students with tasks of this type to encourage them to independently search for knowledge and develop skills such as argumentation and proof. Based on an approach based on the investigation of Chava's theorem, students learn to use mathematical knowledge as a tool to solve complex problems and formulate and test their ideas.

Given a triangle ABC . AD_1 , BD_2 , CD_3 – symmedians (Figure 3).

Prove: $AD_1 \cap BD_2 \cap CD_3 = E$.

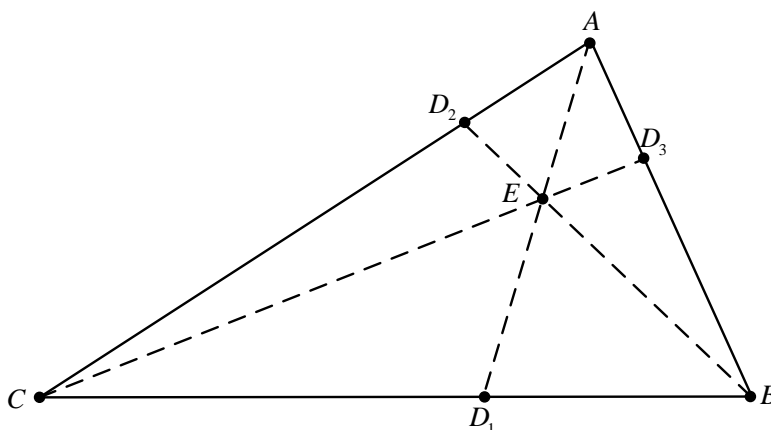


Figure 3. A triangle for proving Cheva's theorem

Source: developed by the author



Cheva's theorem and the properties of the median will be applied using the proof method: the median divides the side to which it is drawn into parts proportional to the squares of the corresponding sides.

If AD_1 , BD_2 , CD_3 are symmedians, then:

$$\frac{BD_1}{D_1C} = \frac{c^2}{b^2}, \frac{CD_2}{D_2A} = \frac{a^2}{c^2}, \frac{AD_3}{D_3A} = \frac{b^2}{a^2} \Rightarrow \frac{BD_1}{D_1C} * \frac{CD_2}{D_2A} * \frac{AD_3}{D_3A} = \frac{c^2}{b^2} * \frac{a^2}{c^2} * \frac{b^2}{a^2} = 1 .$$

So, AD_1 , BD_2 , CD_3 intersect at one point:

$$AD_1 \cap BD_2 \cap CD_3 = E .$$

Geometry, one of the fundamental branches of mathematics, has always played a decisive role in forming analytical thinking. Studying the properties of shapes, understanding ratios, and performing geometric constructions develops students' ability for logical analysis, critical thinking, and abstract thinking. Problems containing elements of geometric constructions introduce the concept of proof and the importance of accuracy in mathematical conclusions into students' minds. The importance of geometry lies in its ability to help students visualise and structure their thinking, which stimulates the skills needed to analyse complex problems, perform precise calculations, and solve real-world problems.

A deep understanding of theorems and the ability to apply them to solve specific problems open the door for students to independent learning. Working with mathematical problems that require the application of Cheva's theorems, properties of symmedians, or other theoretical concepts helps students develop the ability to formulate hypotheses, plan proofs, and solve problems with multiple paths to a solution. By immersing students in the discovery process through independent work, tasks become an essential tool in developing student autonomy, diverting from passive memorisation to active engagement.

Table 1: Traditional vs. Independent Learning Comparison

Aspect	Traditional Teaching	Stimulating Independent Learning
Role of Teacher	Directive, primarily lecturing and demonstrating problem-solving methods.	Facilitator, guiding students to explore and construct knowledge through inquiry-based methods and self-directed activities.
Mathematical Competence Development	Focused on procedural fluency and memorisation of formulas and theorems, including the properties of symmedians, antiparallels, and isogonal lines in geometry.	It encourages conceptual understanding, critical thinking, and the application of mathematical concepts to real-world problems. This includes exploring the significance of Cheva's theorem and the Lemoine point through project-based learning.
Engagement and Motivation	Often reliant on teacher-centred approaches that may not fully engage all students.	It utilises strategies that promote autonomy, relevance, and a sense of competence, such as allowing students to choose their projects or problems related to symmedians or the Lemoine point.



Assessment	Typically focuses on the correctness of solutions and procedural knowledge during examinations, including traditional problems on antiparallels and isogonal lines.	It emphasises formative assessment, reflection, and feedback, encouraging students to understand their learning processes and mistakes, such as misconceptions about Cheva's theorem.
Use of Technology	Limited use, mainly for demonstration purposes.	Integrates technology to explore mathematical concepts dynamically, such as using software to investigate properties of geometric figures like symmedians and the Lemoine point.

Source: developed by the author

Using the latest digital technologies brings the concept of symmedians and other mathematical theories to an entirely new level. Interactive software that allows students to manipulate geometric objects in virtual space makes learning extremely exciting. Students interact with mathematics like never before, exploring the properties of shapes and the operation of theorems in a virtual environment that expands the horizons of their understanding and opens the way to innovative solutions. Using digital tools for visualisation and experimentation allows students to see mathematics as a living, dynamic subject capable of adaptation and change.

Modern digital platforms and tools, such as GeoGebra, Desmos, and Khan Academy, have revolutionised how mathematics is taught and taught, offering teachers and students various opportunities to experiment and explore. Thanks to these resources, students can visualise complex mathematical concepts and experiment with different parameters, contributing to a better understanding of the material. For example, GeoGebra offers interactive capabilities used by millions of students and teachers worldwide, enabling the creation of dynamic mathematical models that can be instantly adapted and modified, promoting deep understanding and long-term retention of mathematical concepts. The Desmos platform, with its high-quality graphing calculators, is used by students to solve problems graphically, and it also allows teachers to create interactive workshops that increase student engagement and understanding.

Discussion

The issue of the development of mathematical competence is widely discussed among scientific publications. Sturmfels et al. (2024) confirm the results of their own research on the teacher's critical role in the transition from traditional teaching methods to approaches that stimulate independent learning. The present study differs from Jumianti et al. (2021), where the emphasis is on learner autonomy without significant attention to the role of the teacher. In the work of Jafarbiglu and Pourreza, (2023) it was found that involvement in the understanding of the concepts of the symmedian of a triangle and Cheva's theorem significantly increases students' ability to think critically. The results themselves resonate with the findings of Aichinger and Rossi (2023) but suggest a more systematic approach to implementing



mathematical concepts in the educational process. The use of digital technologies to support self-directed learning was discussed in the study by Lubold et al. (2023), which pointed to the potential for improving the quality of education. This research complements the hypothesis of Charoenwong et al. (2023) by emphasising the importance of pedagogical support and mentoring from the teacher. According to Shilpa Gopinath (2024), the challenges of independent learning are related to integrating innovative methods in the educational process due to the need to update the digital infrastructure. Based on the results of the study of Kuzle, (2023), the conclusion is confirmed that personalised learning and digital platforms can significantly improve the learning process. According to Moyano et al. (2020), to achieve the maximum effect, it is necessary to ensure the teacher's active participation in solving problems using non-standard methods. Zhao et al. (2024) focus on the potential of modern technologies for developing students' mathematical skills in geometry, where the key is to find the optimal balance between technological innovations and pedagogical methods. Therefore, the question of balance will allow us to realise students' potential to the maximum and prepare them for the challenges of the modern world, which requires further research.

CONCLUSION

Thus, in studying the teacher's role in developing his mathematical competence and students, it was established that transitioning from traditional teaching methods to strategies that stimulate independent learning is vital. Using concepts such as the symmedian of a triangle, antiparallel, isogonal lines, Cheva's theorem, and the Lemoine point have proven practical tools for developing spatial imagination and analytical skills. Applying mathematical concepts in the educational process allows students to learn mathematical material and develop skills necessary for understanding more complex mathematical structures and concepts. The innovative approach encourages independent learning and promotes the formation of students' ability to discover mathematical patterns and solve educational tasks independently.

Among the challenges and problems that the educational process may face in strengthening mathematical competence, the issues of constantly updating educational content and improving competencies stand out. There is a wide range of differences in the levels of mathematics learning among students, which creates difficulties for teachers who seek to meet the needs of each individual. Global challenges affect the quality of education through the internationalisation of education and access to relevant resources. Their solution requires a systematic approach to interaction with parents, educational institutions, and authorities to create equal learning opportunities for all students.

In connection with the identified problems, recommendations and necessary measures include developing and implementing specialised programmes to improve the professional competence of teachers. Modern teachers can conduct specialised training activities by participating in workshops, online courses and conferences. Investments in digital technologies and resources provide students and teachers with access to interactive platforms for visually learning mathematics. It is advisable to involve students in authentic research projects, which include the solution of complex mathematical problems, as a result of which the motivation to study will appear. Participation in international educational programmes and projects and exchanging



experiences with colleagues from different countries will significantly improve mathematics teaching methods and stimulate independent learning among students. The novelty of this work lies in its integration of symmedian research into the teaching of mathematics, using it as a tool to develop students' spatial perception and analytical thinking, thereby offering a fresh perspective on enhancing mathematical competence.

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