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Enhancing Student Mathematical Proficiency through Planimetry and Digital Technologies

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ABSTRACT: Active societal development drives the dynamics of educational approaches to acquiring fundamental knowledge and skills by learners. The necessity of integrating innovative pedagogical technologies into mathematical curricula is prompted by changes in the general processes of perception and thinking among students, as well as the need to adapt educational strategies to these dynamics. The research assesses the impact of planimetric methods and technologies on geometry learning. The research methods used were analysis of literature, pedagogical experiments, and questionnaires of students and teachers. The outcome revealed that incorporating digital technologies and interactive activities enhances the learning of geometrical concepts and boosts students' interest. The results also indicated that the group of students who underwent instructions in line with the study's research hypotheses provided higher solutions to geometric problems than the control group. Questionnaires supported the positive attitude of students and teachers towards the new approaches, noting the enhanced comprehension of the matter and the enhanced interest in the materials. The practical value of the results is in elaborating the methodological recommendations for using digital technologies and planimetric methods in the educational process, which may help enhance the quality of mathematics instruction. Some of the drawbacks of the current study include the generalisability of the results, which suggests the importance of further optimising the ratio of traditional and new teachings applied in the study. The further development direction of the examined field of study is the fine-tuning of the approaches to using technology-enhanced learning activities to boost the efficiency of the teaching-learning process.

Keywords: Planimetry, Lagrange's formula, Digital Technologies, Interactive Teaching Methods, Mathematical Literacy, Geometric Tasks.

I. INTRODUCTION

This work raises questions and hypotheses needed for this study to improve students' mathematical literacy by integrating planimetric methods and ICT in geometry learning. In particular, it examines the effectiveness of these methods in teaching geometry, enhancing students' understanding and motivation, and increasing their academic achievement.

This section formulates the research questions and hypothesis, briefly overviews the article's content, and stresses its relevance. *Problem Statement:* It explores difficulties existing in present methods of education concerning the topic of planimetry. *Objectives:* This section describes the objectives of the study. *Literature Review:* It summarises current literature and approaches to using planimetric and digital technologies in education. *Methodology:* It outlines theory, teaching innovation, and experimentation, in addition to the surveys conducted in the study. *Research Results:* This section explains the results of the experiments and the surveys carried out mathematically and statistically. *Conclusion:* It reprises the presented studies, showing their applicability and the advantages and disadvantages of further studies.

The significance of this study is based on the fact that gaps detected in the literature need to be filled by knowledge on how teaching planimetry with the use of digital technologies and interactive approaches can be enhanced. As Batsurovska et al. [1] and Glenn et al. [2] reveal, digital media promotes interactive learning environments and increases students' interest and knowledge. However, there is still a gap regarding the unified methods incorporating these applications for improving mathematical literacy into traditional teaching methods.

The originality of this study lies in elaborating and validating a complex procedure that combines traditional techniques, planimetric in this case, with modern technologies like virtual laboratories and other educational applications, including GeoGebra and Desmos. Thus, whereas this exercise-based pedagogy research does suggest definite benefits to the spatial ability and motivation of students, this study offers direct quantitative proof that these integrated methods prove successful at enhancing the learners' geometric problem-solving skills as well as their interest levels if the pedagogical experiments and polls are conducted and completed. The findings could be helpful to universities, schools, and policymakers in general who strive to improve the quality of mathematics instruction through more effective teaching methods.

Modern education requires the implementation of innovative approaches to teaching that promote the development of students' mathematical literacy and critical thinking [3]. One such approach is using planimetric methods in education, which allow an understanding of geometric concepts and their practical application. Despite significant achievements in this field, unresolved issues remain regarding integrating digital technologies and interactive methods into the planimetry teaching process. Understanding how these methods affect material comprehension and student motivation is essential. Research shows that digital technologies and interactive teaching methods significantly improve students' understanding and engagement. For instance, Batsurovska et al. [1] note that digital media and technologies in the educational environment enhance students' preparation, increasing their competencies and interest in learning.

Similarly, Glenn et al. [2] analyse the impact of gratitude-based pedagogy on mathematics education and conclude that this approach fosters a deeper understanding of the material. At the same time, the methods of organising a planimetry course for future mathematics teachers, described by Dyupina and Nevzorova [4], demonstrate that using SPOC (Small Private Online Courses) facilitates more effective mastery of planimetric concepts. Research by Lestariningsih et al. [5] shows that developing tasks to enhance mathematical literacy enables students to represent better and solve geometric problems, which is crucial for their further academic and professional growth. Therefore, there is a need for further research and development of methodologies that effectively integrate planimetric methods and modern digital technologies into the teaching process. It will improve students' mathematical literacy and prepare them for successfully applying this knowledge in real life [6].

This study aims to investigate and demonstrate the effectiveness of using the basic concepts of planimetry in teaching methods to develop students' mathematical literacy. The following are considered as the main objectives of this work:

1. To analyses the theoretical foundations and modern approaches to teaching planimetry in the school curriculum.
2. To investigate the impact of using planimetric methods on students' mathematical thinking and skills development.
3. To develop methodological recommendations for the application of formulas and theorems of planimetry to solve geometric problems.
4. To show the practical application of Lagrange's formulas and other key planimetric concepts in solving problems of different levels of complexity.
5. To evaluate the proposed methodology's results based on pedagogical experiments and surveys.

II. LITERATURE REVIEW

Theoretical framework of planimetry and contemporary methods of teaching. Technological education and the use of communicative processes, mostly involving computer-based activities, have received much attention in the recent past. Batsurovska et al. [1] stress the possibilities of digital media and technologies to affect the development of competencies among bachelor's students in the context of the digital learning setting. Dyupina and Nevzorova [4] explain how such a continuing education model as Small Private Online Courses (SPOCs) can promote the improvement of future mathematics teachers' planimetric competencies. Similarly, Glenn et al. [2] contextualise gratitude-based pedagogy and its effects on teaching, specifically in mathematics, which indicates that new-age methodologies might enhance appreciation and comprehension. The collective evidence suggests that contemporary methodologies for utilising digital technologies facilitate a notable intensification of students' engagement and comprehension of the subject matter of planimetry.

Increasing the level of development of mathematical literacy and critical thinking. The creation of mathematical literacy due to undergoing planimetry has been under investigation in some works. Gradini & Firmansyah [7] focus on culturally appropriate learning contexts and their effect on the level of numeracy; there is attention to the cultural contingency of instruction. Eugeny et al. [8] support the practical approach in which different aspects of mathematical knowledge are interrelated, as well as France and Machaba [9], who stress the importance of appropriate strategies while developing mathematical literacy. Jill [10] and Jihyun and Yeajin [11] argue that different methods are needed. All these studies emphasise the necessity of enhancing the students' mathematical literacy, but the standpoints on the desirable approaches to this aim differ slightly.

Some of the things that can dictate the approaches in the teaching and learning of mathematics are the psychological and pedagogical factors. Awareness of the psychological features and the methodologies of teaching mathematics is essential in education. Cognitive load management is examined by Andes et al. [12] in the context of learning MathAILT, and the authors identify it as a critical component. Wesna et al. [13] use a Reciprocal Teaching model to assess the students' independent learning and mathematical literacy and find it helpful; Maryani and Widjajanti [14] argue that contextual learning methods should be used to enhance students' mathematical literacy. To this effect, Orin [15] and Malasari et al. [16] have urged that psychological aspects and meeting and interacting with learners should be considered to facilitate learning. These studies show various ways the psychological and pedagogical issues in teaching mathematics have escalated.

Teaching method application and assessment in the classroom. Many theoretical and empirical findings on using interactive and project-based learning approaches in teaching planimetry are available. Weng [17] examines the concept of mathematical literacy in the context of designing and establishing a virtual factory environment, where the use of various learning aids is valuable and effective in facilitating pupils' work. Sangpom et al. [18] concentrate on developing higher-order mathematics thinking and problem-solving in classrooms, and Bevz [19] on the methodological overview of triangle geometry. In line with this notion, Oliinyk et al. [20] moot how STEM education helps prepare future engineers and the practical implementation of helpful hands-on learning strategies to increase students' interest in geometric concepts. Altogether, these studies imply that demonstrative teaching practices and assessments of the various approaches are necessary for a successful educational process.

Some silences exist [21; 22] concerning integrating the aforementioned digital technologies and the subsequent planimetric methods. The existing literature creates a gap since most studies address specific facets of the digital or traditional media methods of conducting surveys without a proper blend of both. This research intends to meet this gap by systematically designing and validating a teaching technique that combines planimetric methods and new technologies into an effective teaching strategy for geometry.

The analysed literature highlights numerous similarities and differences in scholars' interpretations of the subject. Again, as seen from the literature, there is agreement on the advantages of adopting and using digital technologies and incorporating interactive approaches in the delivery of instruction that can increase students' interest and mastery of lesson content. Nevertheless, controversy arises regarding the appropriate means for incorporating such technologies into typical teaching methodologies. Some scholars recommend an electronic technique, while others propose a hybrid technique using conventional and electronic methods. Therefore, this research plan adds to this discussion by establishing how much a blended approach works.

Teleologically grounded and based on the outlined theoretical conceptualisation and the identified research gaps, the following section of the paper describes the method used in this study to assess the efficacy of incorporating planimetric methods and digital technologies into geometry teaching. Hence, with the help of theoretical considerations, pedagogical experiments, and questionnaires, this work presents a detailed picture of these methods' effects on students' achievement.

III. MATERIAL AND METHOD

This work uses theoretical analysis of concepts, pedagogical experiments, and questionnaires to assess the extent of cognition of the combination of planimetric methods and other digital tools used in geometry teaching. The efficacy of these methods is supported by the fact that SA provides a broad perspective on the researched objectives and issues. The theoretical framework can be used as the background for the discussion of the current state of the scientific observations and the search for the gaps; moreover, the pedagogical experiments can serve as evidence of the efficiency of the suggested approaches. In this sense, surveys supplement these approaches by offering responses from the students' and teachers' perspectives, offering a more comprehensive understanding of the research issue.

1. DATA COLLECTION

The theoretical analysis concerned the modern scientific literature and research on planimetry teaching approaches, digital technologies, and interactivity. Some of the essential terms and definitions that were made and used in the study include planimetry, digital technologies, interactive teaching methods, and mathematical literacy. This step was critical for drawing the theoretical framework and outlining the modern approaches to teaching planimetry.

2. RESEARCH DESIGN

The described pedagogical experiment took place in the teaching groups in which different methods of planimetry teaching were used. In this study, a sample of students was utilised, and they were grouped into an experimental and control group. The experimental group used digital technologies and interactive approaches, including virtual labs, education platforms (GeoGebra, Desmo) and project tasks. In order to guarantee the statistical soundness and validity of the comparison, a total of 15 students were selected from each group, ensuring a robust sample size for the comparative analysis.

For their comprehension, various problems and questions in geometry and trigonometry were presented to the students; such problems involve using the Pythagorean theorem, the sine rule, Heron's formula, and Lagrange's formula. The experimental tasks' degree of reliability and validity was achieved with experts' consultation and piloting the tasks so that they directly measured student performance.

In order to ensure the structured and algorithmic nature of the study, a particular stage of its conduct was envisaged:

1. Survey. Students' and teachers' perceptions of the new teaching methods were assessed via a survey. The survey's ranking questions referred to readiness to study geometry with the help of Information and Communication Technology facilities, understanding of skill improvement, the planimetric formulas' easy comprehensibility, and the final learning satisfaction level. The reliability of the survey items was determined using Cronbach's alpha, while the validity of the survey instrument was checked by conducting an exploratory pilot survey.
2. Several measures were implemented to guarantee the reliability and validity of the research instruments.
3. Pilot testing: The experimental tasks and survey instruments were pre-tested among a small sample of students and teachers to ensure all problems were noted and solved.
4. Expert review: The experimental tasks and survey questions underwent content validity through the assessment by subject matter experts.
5. Statistical analysis: Cronbach's alpha confirmed internal consistency, with a cutoff of 0. 7 indicating acceptable reliability. Factor analysis also confirmed the validity of the instruments.

A significant limitation in the pedagogical experiment is the students' pre-skill level, which can cause differences that can influence the results. In order to mitigate the impact of this variable, the students were randomly assigned to either the experimental or control groups, ensuring that the groups were comparable. The last weakness that could be identified is the problem of validity inherent in survey data in which responses are self-provided. This was done by avoiding people's identification and contributing to people being as truthful as possible.

The selected sample was chosen in a single variant, by random method, comprising 30 students, 15 of whom formed the experimental group, while the other 15 constituted the control group. This sample size was deemed justified in light of practical realities that must be considered while ensuring sufficient power. The sample included diverse students to enhance the applicability of the study to the population under real conditions. The students' demographic data at enrolment and previous academic records were also retained to minimise confounding variables. The general population (the basis of the sample) is represented by learners in the vector of mathematical knowledge.

In conclusion, the selected methods offer a solid research approach for assessing the actors' learning outcomes of planimetric methods and digital tools in teaching geometry. Thus, through theoretical reflection, pilot practices, and questionnaires, this work seeks to provide a multifaceted understanding of the effects of such innovative pedagogy on students' learning accomplishments.

IV. DATA ANALYSIS

1. THEORETICAL DATA ANALYSIS

In order to gain a deeper understanding of the theoretical foundations of the teaching planimetry plan, it is necessary to examine the underlying principles.

Historical development of planimetry. Studying the historical aspects of planimetry's emergence and development, including the contributions of scientists such as Euclid, Apollonius, and others, aids in contemporary teaching.

Basic concepts and axioms. Consideration of basic concepts (point, line, plane), the axioms of Euclidean geometry, and their application in the study of planimetry. Introduction to fundamental theorems and formulas, such as the Pythagorean theorem, properties of angles, circles, and polygons.

Proof methods in planimetry. Studying methods of direct and indirect proof and the use of auxiliary constructions to solve geometric problems [23].

Modern approaches to teaching planimetry include interactive technologies and digital resources, which contribute to a deeper understanding of geometric concepts and the formation of spatial thinking skills. Computer programs and mobile applications allow for the visualisation of complex geometric constructions, facilitating their study and analysis. In Table 1, we will consider the modern approaches to teaching planimetry.

Table 1. Modern Approaches to Teaching Planimetry

Approaches	Description
Interactive teaching methods	Use of digital technologies. Implementation of interactive whiteboards, computer programs and programs for visualising geometric objects and constructions. Online resources and platforms. Educational platforms such as GeoGebra, Desmos, and others can be used to conduct virtual laboratories and interactive lessons.
Project-based learning and research activities	Design tasks. The learning process includes project work, where students can apply planimetric knowledge to solve practical problems, such as designing a schoolyard and creating architectural models. Research work. Encouraging students to conduct independent research and develop and defend research projects on topics related to planimetry.
Differentiated approach	Individual learning trajectories, a personalised approach to learning that considers students' level of training and interests, and the creation of individual curricula that include basic and advanced topics in planimetry. Group work and collaboration. Organising group work to solve complex problems contributes to developing cooperation skills and collective thinking.
Game and competitive methods	Geometry games and quizzes. Introducing game elements into the learning process to increase motivation and interest in the subject. Use of quizzes and competitions to consolidate knowledge. Olympiads and tournaments. Students participate in mathematical Olympiads and tournaments in geometry, which stimulates them to study planimetry in-depth and develop analytical skills.
Meta-subject approach	Connection with other subjects. Planimetry integrates with other disciplines, such as physics, art, and history. For instance, it studies architectural structures from a geometric point of view and uses geometric methods in physics to solve problems in mechanics.

These theoretical and modern approaches to teaching planimetry aim to develop students' deep understanding of the subject, enhance critical and analytical thinking, and prepare them to successfully apply mathematical knowledge in real life. Using planimetric methods in education significantly impacts the development of students' mathematical thinking and skills. In order to gain a comprehensive understanding of the impact in question, it is necessary to consider the various aspects that contribute to it:

a. Development of logical thinking and analytical skills. Planimetry contributes to the development of logical thinking since solving geometric problems requires a clear following of logical steps, construction and proof of theorems. Students learn to:
- analyse the conditions of the task,
- choose the proper methods and approaches for solving the problem,
- formulate and prove hypotheses based on known theorems and axioms.

b. Mastering spatial representations. Geometry tasks develop spatial thinking as they require the representation of three-dimensional objects and their interactions. Students:
- develop the ability to operate with geometric shapes mentally,
- learn to represent and analyse spatial relationships between objects,
- develop visualisation skills that are useful for understanding and solving complex problems.

c. Developing problem-solving skills. Solving planimetric problems develops problem-solving skills. Students:
- learn to identify critical elements of problems and apply appropriate methods to solve them,
- develop the ability to decompose complex problems into more straightforward steps and solve them sequentially,
- acquire critical thinking and analytical skills.

d. In-depth understanding of mathematical concepts. The study of planimetry contributes to an in-depth understanding of mathematical concepts. Students:
- master basic concepts and theorems that are the basis for further study of mathematics,
- learn how to apply theoretical knowledge in practice, which helps to consolidate the material,
- develop skills of generalisation and transfer of knowledge to new situations.

e. Increase motivation and interest in mathematics. Interactive and visual planimetry teaching methods can significantly increase students' motivation and interest in mathematics. Students:
- are more actively involved in the learning process through the use of digital technologies and interactive methods,
- show more interest in solving problems when they see the practical application of theoretical knowledge,
- enjoy the satisfaction of successfully solving complex problems, stimulating further learning.

f. Development of self-control and independent learning skills. Working with geometric problems develops self-control and independent learning skills. Students:
- learn to check their solutions and find mistakes,
- develop skills in self-organisation and planning of the learning process,
- become more independent in learning new topics and completing tasks.

g. Preparation for competitions and contests. The study of planimetry helps prepare for mathematical competitions and contests, which contributes to developing competitive skills and self-confidence. Students:
- gain experience in solving problems of increasing complexity,
- develop strategic thinking and the ability to think outside the box,
- get the opportunity to demonstrate their skills at the highest level.

Using planimetric methodologies in educational settings facilitates the advancement of mathematical reasoning and abilities. It fosters a comprehensive and profound comprehension of the subject matter, the foundation for future success in mathematics. It is crucial to consider methodological recommendations for applying formulas and theorems of planimetry to solve geometric problems.

1.1 Application of the Lagrange's formula to calculate the length of a bisector

Problem: Find the length of the bisector AL_1 of triangle ABC given the known sides $AB=c$, $AC=b$, and $BC=a$ using Lagrange's formula.

Lagrange's formula: $AL_1^2=AC\cdot AB-CL_1\cdot L_1B$ or $l_a^2=bc-b_1c_1$

Recommendations:

- First, find the values of $b1$ and $c1$ (the lengths of the sides of the triangle divided by the side BC and the point where the bisector intersects).
- Substitute the known values into the formula and calculate the bisector length.

1.2 *Use of central and inscribed angles*

Problem: Given triangle ABC, find the radii of the circles circumscribed by triangles ACL1 , ABL1 and ABC and prove that the radii are equal.

Theorem:

If $\gamma 1$ (O1, R1), $\gamma 2$ (O2, R2) and $\gamma 0$ (O, R) are the radii of the circumscribed circles, then (OO1)2 =(OO2)2=R2 −R1·R2

Recommendations:

- Draw the required circles and determine their radii.
- Use the properties of central and inscribed angles to prove that the radii are equal.
- Solve the problem by substituting values and using known formulas.

1.3 *Solving problems using the formula of three*

In the context of planimetry, the triple formula often refers to the theorem about the product of the lengths of the sides of a triangle intersected by a bisector.

Problem: Prove the equality of the products of the side segments of a triangle AB=AL1·AW1

Formula of three:

AC·AB=AL1·AW1

Recommendations:

- Use the similarity of triangles to find the ratio of the sides.
- Plug the values into the formula and work out the proof by looking at the equality of the sides and angles.

1.4 *Application of Lagrange's formula to solve complex problems*

Problem: The bisector AL1 is drawn in triangle ABC. Points O1, O2, and O are the centres of the circles circumscribed around the triangles ACL1, ABL1, and ABC, respectively. It is required to prove that (OO1)2=(OO2)2=R2−R1·R2.

Recommendations:

- Draw the circles and determine the radii.
- Use Lagrange's formula for triangle AO1O2 and perform the necessary calculations.
- Prove equality by using the properties of inscribed and central angles and segment equality.

1.5 *Using the formula to calculate the length of a chord*

Problem: In triangle ABC, find the lengths of the chords OO1 and OO2 through the radii of the circumcircles and the angles of the triangle.

Formula:

OO1 =R·cos(α)

OO2 =R·cos(β)

Recommendations:

- Find the angles α and β of the triangle.
- Substitute the values into the formula and calculate the lengths of the chords.
- Use geometric properties and theorems to support your results.

These methodological recommendations will help effectively apply planimetric formulas and theorems in the learning process and develop students' skills in solving geometric problems of varying complexity. Planimetry is crucial in developing students' mathematical skills, offering various methods for solving geometric problems. The practical application of formulas, such as Lagrange's, effectively solves problems related to calculating the lengths of sides and angles of triangles. Using planimetric concepts, such as the inscribed and central angles theorem, helps students better understand the structure and properties of geometric figures. In solving planimetric problems, applying different approaches, including proofs and visualisations, is essential. In order to illustrate the practical application of these formulas and theorems, it is necessary to consider several examples of varying complexity.

Formulas in geometry are an effective technique for solving problems. For instance, to find the length of a triangle's bisector given its three sides, the so-called Lagrange's formula (I) is used. We propose considering the following Lagrange's formula, which we will call "Lagrange's formula II." We will demonstrate the application of "formula II" to solving problems. Lagrange's formula (hereafter referred to as "Lagrange's formula I") (see Figure 1) has been known to humanity since ancient times for calculating the length of a triangle's bisector:

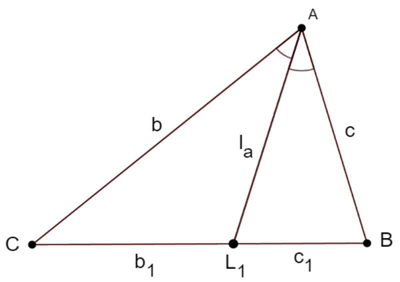

FIGURE 1. Lagrange's formula

$(AL_1)^2 = AC \cdot AB - CL_1 \cdot L_1B$ (1),
or $(l_a)^2 = bc - b_1c_1$

Proof of the Lagrange's formula (I) (see Figure 2):

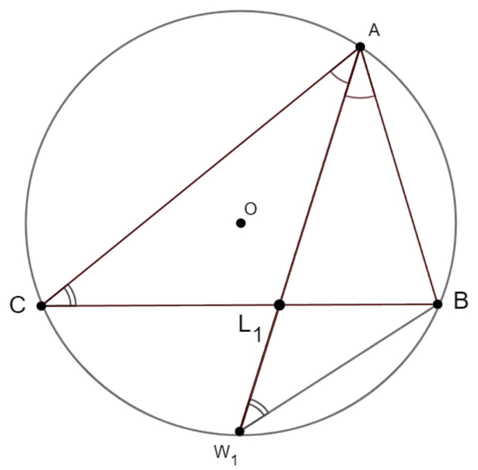

FIGURE 2. Proof of the Lagrange's formula (I)

1) $\triangle ACL_1 \sim \triangle AW_1B$

$$\frac{AC}{AL_1} = \frac{AW_1}{AB}$$

$AC \cdot AB = AL_1 \cdot AW_1$ (2)

2) $AC \cdot AB = AL_1 \cdot (AL_1 + L_1W_1)$

$AC \cdot AB = (AL_1)^2 + AL_1 \cdot L_1W_1$

$AC \cdot AB = (AL_1)^2 + CL_1 \cdot L_1B$

Alternatively,

$(AL_1)^2 = AC \cdot AB - CL_1 \cdot L_1B$

Proved.

Turning to formula (2):

$$AC \cdot AB = AL_1 \cdot AW_1$$

In the literature of Kushnir [24], it (2) is called the 'trinity formula'. Here, we write it differently:

$$AC \cdot AB = (AW_1 - L_1W_1) \cdot AW_1$$

The sum of $AW_1 \cdot L_1W_1$ is equal to $(CW_1)^2$. Proof (see Figure 3):

$$AC \cdot AB = (AW_1)^2 - AW_1 \cdot L_1W_1 \quad (3)$$

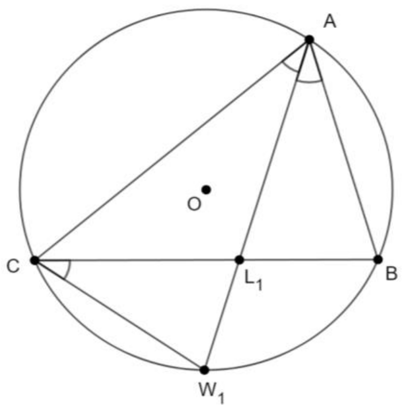

FIGURE 3. Proof of the «trinity formula»

$$\triangle CW_1A \sim \triangle L_1W_1C$$
$$\frac{CW_1}{AW_1} = \frac{L_1W_1}{CW_1}$$
$$(CW_1)^2 = AW_1 \cdot L_1W_1 \quad (4)$$

Proved.

Therefore,

$$AC \cdot AB = (AW_1)^2 - (CW_1)^2 \quad (5)$$

The formula (△△) will be called the "Lagrange's II".
$CW_1 = BW_1$ (equal arcs constrain equal chords).

It is important to note that:

$$CW_1 = BW_1 = IW_1 \quad (6)$$

(where I is the centre of the inscribed circle in triangle ABC , the point of intersection of the bisectors, the centre) (see Figure 4). Proof:

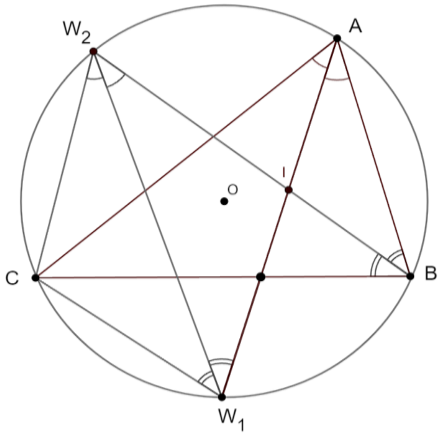

FIGURE 4. Proof of the Lagrange's formula (II)

$\triangle W_2CW_1 = \triangle W_2IW_1$ (W_1W_2 – common side, $\angle CW_2W_1 = \angle IW_2W_1$; $\angle CW_1W_2 = \angle IW_1W_2$ adjacent angles).
From the equality of triangles W_2CW_1 and W_2IW_1, the equality of sides follows:

$$CW_1 = IW_1$$

Proved.

We have $CW_1 = BW_1 = IW_1$, the shamrock theorem initially developed by Kushnir [24].
Thus, the "Lagrange formula II" can be written differently:

$$AC \cdot AB = (AW_1)^2 - (IW_1)^2 \quad (7)$$

Problem on the application of Lagrange's formula II.

The bisector AL_1 is drawn in triangle ABC. The points O_1, O_2, *and* O are the centres of the circles described around the triangles ACL_1, ABL_1, and ABC, respectively. Radii R_1, R_2, R of these circles. Prove that $(OO_1)^2 = (OO_2)^2 = R^2 - R_1 \cdot R_2$ (see Figures 5–12).

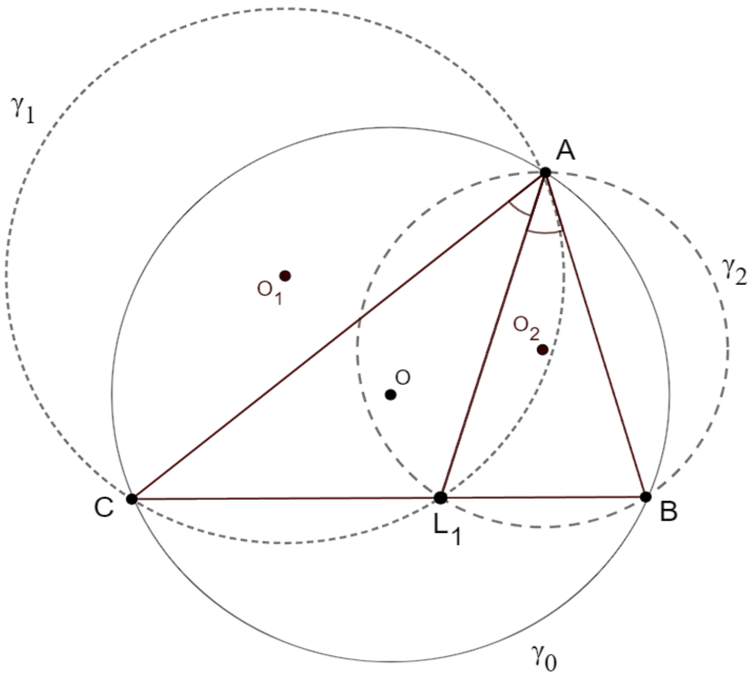

FIGURE 5. Problem on the application of Lagrange's formula II (a)

Given:
$\angle CAL_1 = \angle BAL_1$;
$\gamma_1(O_1;\ R_1 = O_1A)$;
$\gamma_2(O_2;\ R_2 = O_2A)$;
$\gamma_0(O; R = OA)$.
Prove:
$(OO_1)^2 = (OO_2)^2 = R^2 - R \cdot R_2$ provided that $\angle ABC > \angle ACB$

Proved.

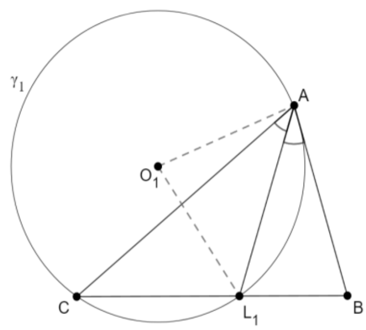

FIGURE 6. Problem on the application of Lagrange's formula II (в)

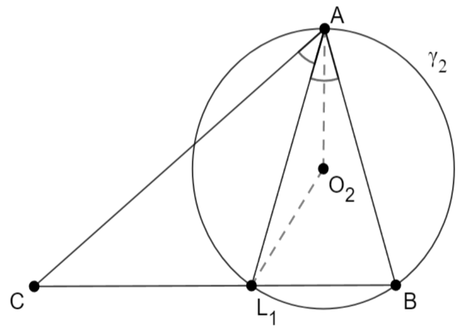

FIGURE 7. Problem on the application of Lagrange's formula II (c)

$\gamma_1(O_1;\ R_1 = O_1A)$;
$\angle O_1AL_1 = 90° - \angle ACB$ of a triangle AO_1L_1.

$\gamma_2(O_2;\ R_2 = O_2A)$;
$\angle O_2AL_1 = 90° - \angle ABC$ of a triangle AO_2L_1

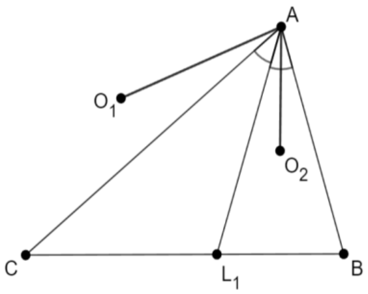

FIGURE 8. Problem on the application of Lagrange's formula II (d)

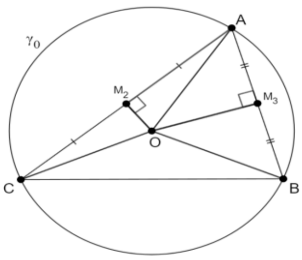

FIGURE 9. Problem on the application of Lagrange's formula II (e)

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$\angle O_1AO_2 = \angle O_1AL_1 + \angle O_2AL_1 = 90° - \angle ACB + 90° - \angle ABC = \angle BAC$

I.

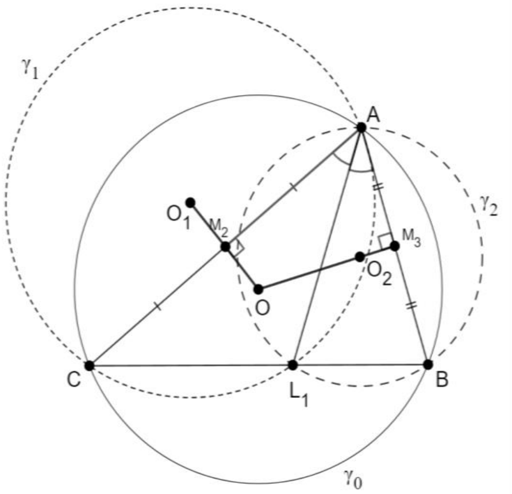

FIGURE 10. Problem on the application of Lagrange's formula II (f)

$\gamma_0(O;\ R = OA)$;
$CM_2 = M_2A$;
$\angle AOM_2 = \angle ABC$ of a triangle AOM_2
$AM_3 = M_3B$;
$\angle AOM_3 = \angle ACB$ of a triangle AOM_3
$\gamma_1 \cap \gamma_0 = AC$;
$OO_1 \perp AC$; $\Rightarrow \angle AOM_2 = \angle AOO_1$
$OO_1 \cap AC = M_2$
$\gamma_2 \cap \gamma_0 = AB$;
$OO_2 \perp AB$; $\Rightarrow \angle AOM_3 = \angle AOO_2$
$OO_2 \cap AC = M_3$

Therefore, $\angle O_1OO_2 = \angle AOO_1 + \angle AOO_2 = \angle ABC + \angle ACB = 180° - \angle BAC$.

Therefore, a circle can be described around the quadrilateral AO_1OO_2 because the sum of the opposite angles $\angle O_1AO_2$ and $\angle O_1OO_2$ is 180°.

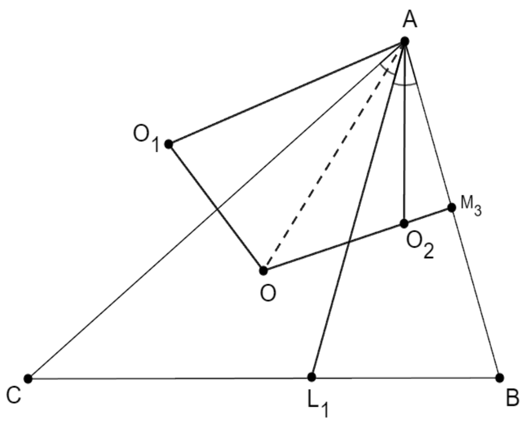

FIGURE 11. Problem on the application of Lagrange's formula II (g)

For triangle AOO_2 angle AO_2M_3 is external:

$\angle AO_2M_3 = \angle AOO_2 + \angle AOO_2$;

So, $\angle AOO_2 = \angle AO_2M_3 - \angle AOO_2$;

$\angle AO_2M_3 = \angle AL_1B$ (with γ_2 as the central circle). Therefore

$\angle OAO_2 = \angle AL_1B - \angle ACB = \frac{\angle BAC}{2}$.

Conclusion: for angle O_1AO_2 AO is a bisector, or $\angle O_1AO = \angle O_2AO$.

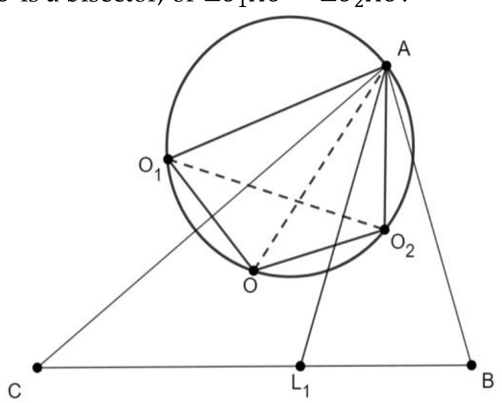

FIGURE 12. Problem on the application of Lagrange's formula II (h)

Thus, the chords OO_1 and OO_2 are equal because they make equal arcs. In the triangle O_1AO_2, the sides O_1A and O_2A are equal to R_1 and R_2, respectively. Apply the "Lagrange's formula II" to the triangle AO_1O_2:

$AO_1 \cdot AO_2 = (AO)^2 - (OO_1)^2$, або $R_1 \cdot R_2 = R^2 - (OO_1)^2$.

Otherwise,

$(OO_1)^2 = (OO_2)^2 = R^2 - R_1 \cdot R_2$.
Proved.

The considered tasks show how planimetric methods can effectively solve complex geometric problems. The practical use of formulas and theorems develops students' mathematical skills and deepens their understanding of geometric concepts. It contributes to the formation of solid skills in logical and analytical thinking, which are necessary for the successful study of mathematics. This material is advisable for use in extracurricular classes, clubs, and olympiads and is recommended for all enthusiastic about geometry.

In order to assess the effectiveness of the proposed methodology for employing planimetric techniques in an educational setting, a series of pedagogical experiments and surveys were conducted among students and educators.

2. PEDAGOGICAL EXPERIMENTS

During the experiments, groups of students taught using the proposed methodology (planimetric methods) were compared with control groups using traditional teaching mesthods. To experimentally compare the quality of planimetry education, students were given tasks involving the use of the Pythagorean theorem, the application of the sine rule, the use of Heron's formula, and the application of Lagrange's formula (Appendix A). The results of the problem-solving are presented in Table 2 and Table 3.

Table 1. Table of Results of Problem Solving in the Experimental Group (Planimetric Methods)

Student	**Pythagorean theorem (Task 1)**	**Sine Theorem (Task 2)**	**Heron's formula (Task 3)**	**Lagrange's formula (Task 4)**	**Total score**
Student 1	**10**	**9**	**9**	**8**	**36**
Student 2	9	8	10	9	36
Student 3	10	10	9	8	37
Student 4	9	9	9	9	36
Student 5	10	9	10	10	39
Student 6	8	10	9	10	37
Student 7	9	10	10	9	38
Student 8	10	8	10	10	38
Student 9	10	9	8	9	36
Student 10	8	9	9	9	35
Student 11	9	9	10	10	38
Student 12	10	10	8	10	38
Student 13	10	10	9	8	37
Student 14	9	9	10	9	37
Student 15	10	8	9	10	37
Average value	9,40	9,13	9,27	9,20	37,00

Table 2. Table of Results of Problem-Solving in the Control Group (Traditional methodology)

Student	**Pythagorean theorem (Task 1)**	**Sine Theorem (Task 2)**	**Heron's formula (Task 3)**	**Lagrange's formula (Task 4)**	**Total score**
Student 1	8	7	7	6	28
Student 2	7	6	8	7	28

Student 3	8	8	7	6	29
Student 4	7	7	7	7	28
Student 5	8	7	8	8	31
Student 6	7	6	7	7	27
Student 7	7	6	8	7	28
Student 8	8	7	7	7	29
Student 9	7	7	7	8	29
Student 10	7	8	9	6	30
Student 11	6	8	7	7	28
Student 12	8	7	8	8	31
Student 13	8	6	7	7	28
Student 14	9	8	7	7	31
Student 15	7	7	9	7	30
Average value	7,47	7,00	7,53	7,00	29,00

The results show that students taught using the planimetric methods demonstrated higher results than those using traditional teaching methods. It confirms the effectiveness of the proposed methodology in developing students' mathematical thinking and skills (Figure 13).

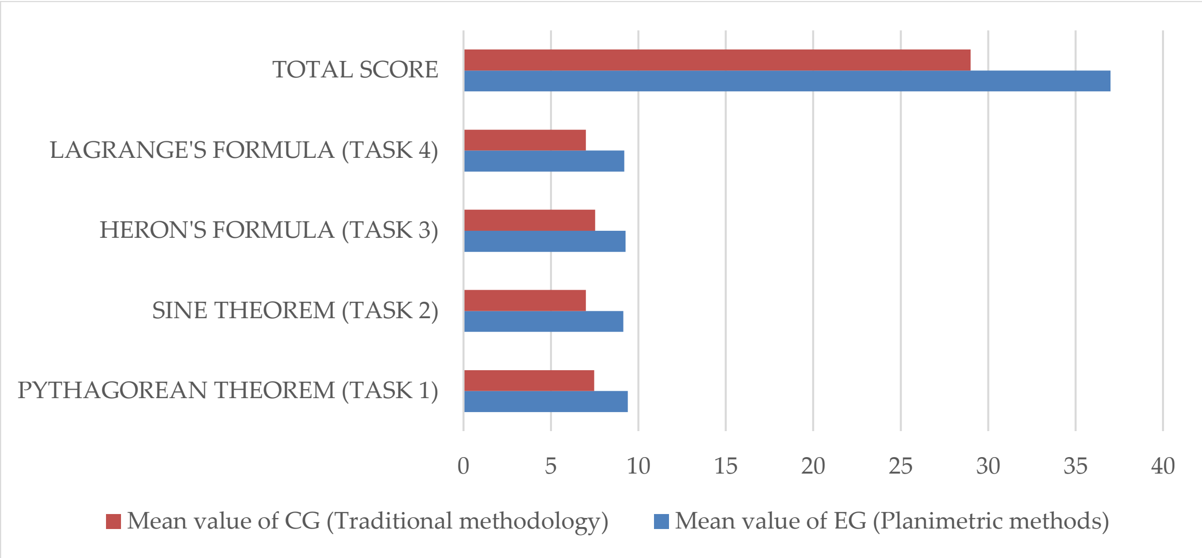

FIGURE 13. Comparative Histogram of the Results of Solving Problems in the Control and Experimental Groups

Students in the experimental group significantly improved in solving geometric problems of varying complexity. The average score increased compared to the control group. The level of understanding of theoretical concepts in planimetry improved, as confirmed by the results of tests and control works (Appendix B). Students began showing more interest in the subject, reflected in the increased homework and lesson activity.

The statistical analysis carried out in this study is crucial to confirm the efficiency of the new teaching approach regarding the planimetric methods and digital tools. In order to compare the students' results about various geometric tasks, the mean values and standard deviations were computed for each group. Subsequently, a T-test was calculated to assess the veracity of the null hypothesis, which postulated that the experimental and control groups would exhibit no statistically significant difference. The obtained t-statistics and p-values are all less than 0. 05 level of significance, so it can be concluded that the observed differences are statistically significant and not just due to chance. Such statistical techniques are very stringent and rigorous, making the results most reliable and valid, underlying the fact that the new methodology contributes a lot to improving students' understanding and problem-solving skills in geometry.

Statistical calculations, Mean value and standard deviation. For each group, we will calculate the standard deviation δ for each type of task using the formula:

$$\delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - x)^2} \qquad (8)$$

To test the significance of the differences between the two groups, we will conduct a t-test:

$$t = \frac{x_1 - x_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad (9)$$

Where x_1 and x_2 are the mean values of the two groups,
s_1 and s_2 are standard deviations,
n_1 and n_2 are sample sizes.
We will perform the calculations and present the T-test for comparing the groups in Table 4.

Table 3. Mean Values and T-test Results for Comparing the Two Groups for the Experimental and Control Groups

Methodology	Experimental group	Control group	t-statistic	p-value
Pythagorean theorem	9,4	7,47	7,15	8.72·10-8
Sine theorem	9,13	7,00	7,79	1.72·10-8
Heron's formula	9,27	7,53	6,56	4.12·10-8
Lagrange's formula	9,20	7,00	8,40	3.88·10-8

The mean values for all methods are significantly higher in the experimental group compared to the control group. The t-test results show that the differences between the groups are statistically significant for all methods, as the p-values are much less than 0.05. It confirms the effectiveness of using planimetric methods for teaching mathematics.

A survey (Appendix C) was conducted among students taught using the proposed methodology to assess their perceptions and satisfaction. The survey results among students are presented as percentages in Table 5.

Table 4. Results of the Student Survey in Percentage Terms

Question	Very interesting	Interesting	Neutral	Not interested	Very uninteresting
1. How interested are you in learning geometry using interactive methods and digital technologies?	60%	25%	10%	5%	0%
Question	Significantly improved	Improved	No changes	Deteriorated	Significantly deteriorated
2. Do you feel your geometry problem-solving skills have improved thanks to the new methodology?	55%	35%	10%	0%	0%
Question	Very easy	Easy	Neutral	Difficult	Very difficult

3. How easily could you understand and apply planimetric formulas and theorems?	50%	40%	10%	0%	0%
Question	Very satisfied	Satisfied	Neutral	Dissatisfied	Very dissatisfied
4. How would you rate your satisfaction with the learning process using the new methodology?	60%	30%	10%	0%	0%
Question	Yes, I would like to	Yes	Not sure	Rather not	No
5. Would you like to continue your studies using similar interactive and digital methods?	65%	25%	10%	0%	0%

There was a noticeable interest in learning, indicating a high level of engagement and motivation among students. An improvement in skills was recorded – 55% of students noted a significant improvement in their skills in solving geometric problems due to the new methodology, 35% noticed improvement, and 10% did not note any changes. No student indicated a skill deterioration, which indicates the methodology's effectiveness. Ease of understanding was also noted during the survey. 50% of students believe that it has become straightforward to understand and apply planimetric formulas and theorems in practice, 40% find it easy, and 10% are neutral. It confirms that the methodology makes the material more accessible and understandable. Students report satisfaction with the learning process. 60% of students are delighted with the learning process, 30% are satisfied, and 10% are neutral. The high level of satisfaction confirms the positive perception of the methodology. There is a desire to continue learning: 65% of students want to continue learning using similar interactive and digital methods, 25% are also interested, and 10% are uncertain. This indicates that most students are willing to continue learning using this methodology.

Overall, Table 5's results demonstrate that students perceive the new methodology positively, are highly engaged, have improved skills, and are satisfied with the learning process.

Teachers who applied the proposed methodology were also surveyed for their opinions and comments (Appendix D). The teacher survey results are presented in Table 6 in percentage terms.

Table 5. Results of the Survey among Teachers in Percentage

Question	**Significantly increased**	**Increased**	**Did not change**	**Reduced**	**Significantly reduced**
1. To what extent do you think the new methodology has increased students' interest in geometry?	70%	20%	10%	0%	0%
Question	Yes, it has improved significantly	Yes, it has improved	No change	Deteriorated	Significantly deteriorated
2. Do you notice an improvement in students' understanding of planimetric concepts and formulas?	65%	25%	10%	0%	0%
Question	Much easier	Easier	No change	Harder	Much harder
3. How much easier is it to explain complex geometric concepts using the new methodology?	60%	30%	10%	0%	0%
Question	Very high	High	Average	Low	Very low
4. Assess the extent to which students are engaged in the	70%	20%	10%	0%	0%

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learning process using interactive methods:					
Question	Yes, definitely	Yes	Probably	Rather, no	No
5. Do you plan to continue using this methodology in the future?	75%	15%	10%	0%	0%

In order to additionally support the interpretation of the presented research results, diagrams are provided indicating the survey data of teachers concerning their attitude to the new teaching method of geometry. Figures 14–16 below are responses to questions on how the implementation of the methodology helped in increasing students' interest in geometry, enhancing understanding of planimetric concepts and formulas, the easiness of explaining complex geometric ideas, students' engagement and their intention in the future to continue being taught using thematic technique.

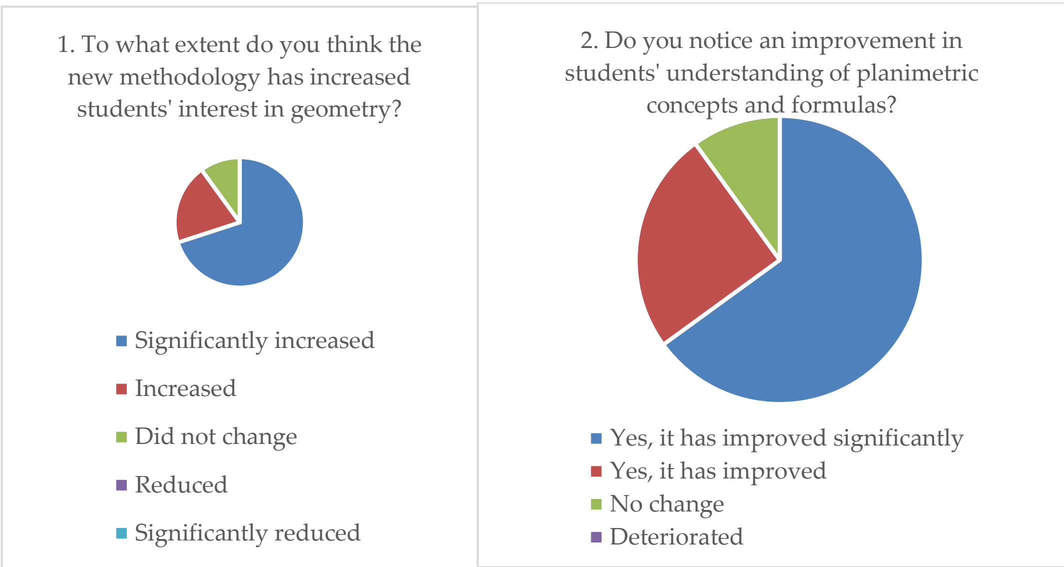

FIGURE 14. Results of the Teacher Survey on the Effectiveness of the New Geometry Teaching Methodology

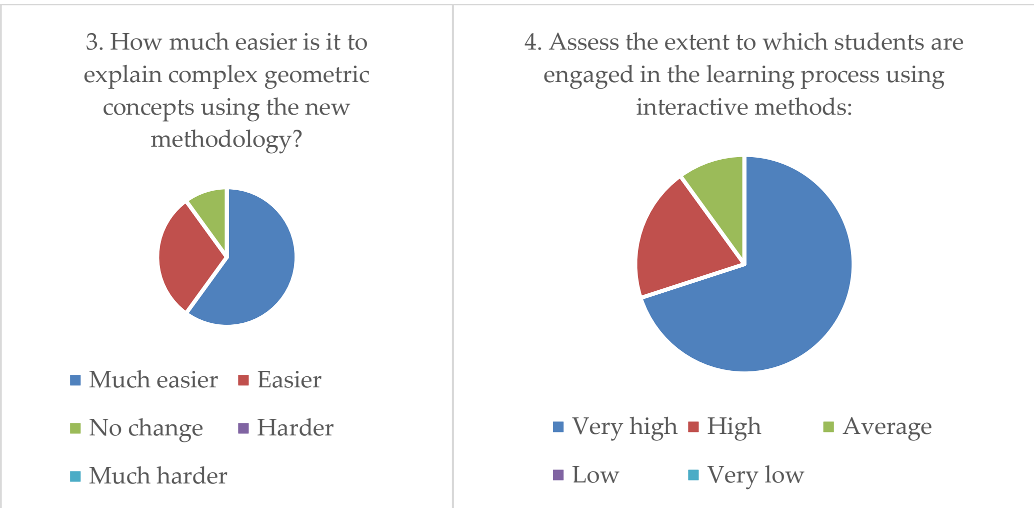

FIGURE 15. Results of the Teacher Survey on the Convenience and Student Engagement with the New Teaching Methodology

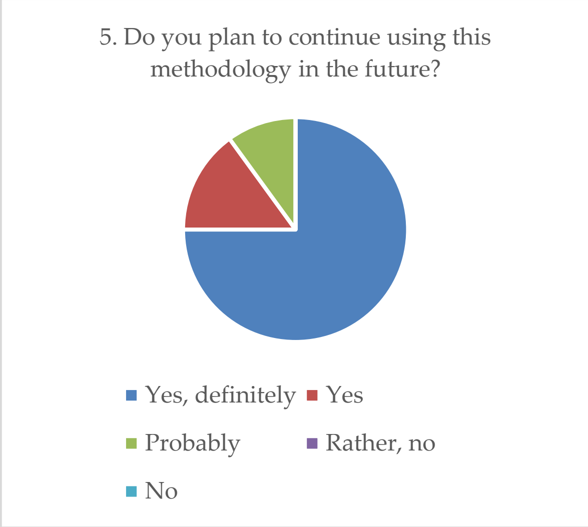

FIGURE 16. Results of the Teacher Survey on Future Plans to Use the New Methodology

Overall, Table 6 shows that teachers positively evaluate the new methodology, noting an increase in students' interest and engagement, improved understanding of concepts, and an intention to continue using this methodology in the future. This indicates that the methodology effectively motivates students, improves their understanding of planimetric concepts, helps them to learn the material better, facilitates the teaching process, and promotes active participation.

The results of the experiments and surveys show that the proposed methodology for using planimetric methods in teaching significantly increases students' mathematical literacy levels, improves their understanding and interest in the subject, and promotes the development of critical skills necessary for successful mathematics learning.

The findings of this study have several important implications for the teaching of mathematics, especially concerning the use of new technologies to enhance geometrical learning.

Enhanced student engagement and motivation. The use of digital tools, particularly the incorporation of interactive activities in the teaching planimetry, was found to enhance student motivation. This was evident from the fact that students who underwent the new methodology had higher interest levels as compared to those who did not. With such an approach, students will be more focused on the topic and develop long-term interests in mathematics, which would benefit their academic success and future learning and application of mathematics.

Improved problem-solving skills. Regarding problem-solving, the experimental group that engaged digital technologies and an interactivity approach performed better than the control group. This points to the fact that incorporating these contemporary techniques improves students' comprehension of such concepts in mathematics. By solving geometric problems with the aid of tools like GeoGebra and Desmos, the learners' analytical and logical thinking is enhanced.

Development of spatial thinking. As the study pointed out, spatial thinking is a crucial concept in the study of geometry. Planimetry and digital support help develop spatial orientation skills and planimetric methods. This is important to establish a relationship between different geometric entities and solve space-related issues, which are common in most STEM subjects.

Positive attitudes towards mathematics. Students and teachers positively perceived the new teaching strategies adopted in schools. Students displayed improved satisfaction with learning, as did teachers' increased ability to explain geometric concepts. This positive perception is important because it can lead to a better perception of mathematics, which could mean a better learning environment and improve the understanding of mathematics.

Methodological recommendations for curriculum development: The study's knowledge and methodological tips can be helpful for curriculum developers. Applying the ICT and interactive approach to learning enhances students' experience and makes the mathematics class more engaging. This blended approach can be adapted according to educational levels and contexts, improving the quality of mathematics education.

Implications for future research. Therefore, the authors list several directions for further research: the long-term effects of using such approaches, the possibilities of applying the suggested teaching methods in other educational settings, and the problem of finding the right proportions between adopting both traditional and new forms of teaching so that the students are not overwhelmed by information. Such future studies can further understand the most suitable ways to adopt technology in mathematics education.

Therefore, incorporating planimetric methods and information technologies into mathematics learning enhances the students' mathematical abilities, interest, and enthusiasm in geometry. The positive findings noted in this study point to the possible advantages of implementing such liberal pedagogy paradigms in teaching mathematics in general and, subsequently, improved performance and understanding of mathematical ideas.

One of the notable discoveries was that even the students who studied with the traditional approach benefitted in problem-solving, though not as much as those in the experimental group. This implies that even marginal alterations in the delivery of education can improve the results that are achieved. In particular, it was noted that using a new methodology facilitated the explanation of complex geometric concepts, which positively affected the student's general perception of the topic. Notably, the students showed a lot of responsibility and autonomy when working with digital technologies. Thus, the results obtained in the given work contribute to the understanding of the need for increased flexibility and the changes in the methods used to teach in order to reach better educational outcomes.

V. DISCUSSION

The implication of the research result in this study is that it has essential ergonomics and practical importance for educators and curriculum developers. It has also been discussed that incorporating planimetric methods and digital technologies helps students improve their knowledge of geometric disciplines and boosts their desire to learn. For teachers, it implies that integrating applications such as GeoGebra and Desmos, as well as teaching activities, can enhance teaching practices' effectiveness. Curriculum developers should bear these findings while developing geometry curricula with the appropriate integration of digital technologies to augment traditional teaching techniques. Expanding methodological recommendations from this study can be useful for educators interested in applying such strategies in practice.

The findings of this study are corroborative of studies undertaken in another related research. Pala et al. [25] and Afni and Hartono [26] stress the importance of digital technologies in the context of students' performance. In the same context, Ariawan and Ardana [27] have also commented on the usefulness of modern methods in the digital environment in consolidating knowledge incorporating micro-theories. Thus, the present work enriches the literature by empirically evaluating a hybrid solution that integrates planimetric techniques with computer-supported tools. This methodical approach helps to align the gaps that previous researchers [30] have proven to the fact that they work only the vital functions of both digital and traditional methods independently.

Researchers Schmid et al. [29] explored the GeoGebra software, with its constructivist approach to teaching, which proves to be a powerful tool for visualizing geometry and preparing for immersion in the educational process using virtual reality (VR) and augmented reality (AR). The authors argue that for educators, such integration of combined geometry lessons using VR and augmented reality through GeoGebra enables teachers to create transformational learning opportunities and fosters a deeper understanding of geometry. These conclusions complement the results of the current study.

Furthermore, Eisenmann et al. [30] demonstrate how technological devices can be integrated into students' use of four selected heuristic strategies in solving mathematical problems: systematic experimentation, introduction of an auxiliary element, analogy, and drawing a solution. The researchers' study represents students' success levels and attitudes toward problem-solving due to learning the four selected heuristic strategies while actively using technological devices. A statistically significant increase in problem-solving success and decreased frequency of 'no answer' cases (where the solver does not start solving the problem) were expected.

This work's outcomes and drawbacks enshrine several potential courses of investigation. First, there is an imperative for a qualitative analysis of the impact of utilising planimetric techniques and digital tools on the learners. Cohort research may yield helpful information about the effects of these approaches on the students' overall mathematical literacy and other related competencies in a longitudinal sense. However, future research must examine the advanced strategy of adjusting between old-fashioned teaching techniques and the use of computerised tools in order not to overload and make the learning process unproductive. Other research that can be conducted in the future can also consider the applicability of these methods to other age and educational levels.

Therefore, this study has the following limitations that other studies can address: There is a glimpse of focus on a small sample, and therefore, the results do not increase the reliability percentage. However, further research should employ a large population sample and encompass a diverse population community to increase the validity of the findings. Another weakness is the variation in students' backgrounds and prior knowledge, which can affect the results. This study minimised the issue by random assignment. However, subsequent studies should aim to undertake more comprehensive controlled experiments on these variables.

VI. CONCLUSION

The findings of this study are highly encouraging for the implementation of planimetric methods and the use of digital technologies in the teaching of geometry. Therefore, this research emanates the following noteworthy implications for mathematics education. It proves that the integration of conventional techniques of planimetry computations with information technologies will help improve learners' comprehension of geometrical notions and stimulate them. Consequently, the decline in the control group's performance demonstrated the feasibility of the blended learning approach advocated for students in the experimental group to perform better in solving geometric problems.

This paper gives a research-based rationale for incorporating ICTs and planimetric techniques to supplement students' learning. They provide specific suggestions to educators about implementing a technical approach in their teaching-learning process. The research discussed interactive techniques as capable of significantly impacting students' interest and participation in geometry. The findings of the teachers and curriculum designers imply that a purposeful incorporation of ICT into mathematics, specifically geometry, would enhance the teaching and learning process. Teachers are advised to incorporate the use of such technologies as GeoGebra and Desmos in their teaching practices. Therefore, curriculum developers should ensure that these elements are incorporated in the development of educational programs to enhance the learning process.

Future research should focus on several key areas to build on the findings of this study:

- Longitudinal studies: One recommendation is to conduct research over a long period to assess the effectiveness of using digital technologies and planimetric methods to improve students' learning retention.
- Diverse educational contexts: Refer to the literature to discover the application and usability of these methods in different age groups and other educational institutions and to determine their generality.
- Optimal integration strategies: Find out how to extend the concept of blended learning, the positive effects of such an approach, and the risks associated with overloading a student's brain.
- Detailed tool analysis: Investigate the effectiveness of particular software and pedagogical practices to determine the influential factors in increasing students' achievements.

Such recommendations should be considered by policymakers when formulating education policies and programs. An important factor would be to guarantee that teachers are prepared to integrate ICTs into their lessons and are given the tools and means to do so effectively.

Therefore, this study's findings highlight the role of adopting both planimetric methods and technology-centred strategies in facilitating geometry education. The specificity derived from outlining concrete methodological recommendations and indicating directions for practical use identifies possible approaches to optimally developing the subject area and promotes further improvement of mathematics education.

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Author contribution

The author, Liudmyla Hetmanenko, was solely responsible for this manuscript's conception, design, data collection, analysis, and writing.

Conflict of Interest

The author declares no conflict of interest.

Data Availability Statement

Data are available from the author upon request.

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Appendix A

Example tasks

Task 1: Using the Pythagorean theorem
Problem: In a right triangle ABC with right angle C, the catenaries AC=3 and BC=4. Find the length of the hypotenuse AB.

Task 2: Applying the sine theorem
Problem: In triangle ABC, angle A is 30^0, angle B is 60^0, and side BC is 10. Find the length of the side AC.

Task 3: Using Heron's formula
Problem: Find the area of a triangle if its sides are a=7, b=8 and c=9.

Task 4: Application of the Lagrange's formula
Problem: In triangle ABC, the sides AB=10, AC=6, BC=8. Find the length of the bisector drawn from vertex A.

Appendix B

Example of a Test for Students in Planimetry

Option 1
1. In a right triangle ABC with correct angle C, the catenaries AC=6 and BC=8. Find the length of the hypotenuse AB.
a) 10
b) 12
c) 14
d) 16
2. Find the area of a triangle with sides a=5, b=6 and c=7 using Heron's formula.
a) 10
b) 14
c) 15
d) 12
3. In triangle ABC, angle A is 45^0, side AB = 10, and side BC = 10. Find the length of the side AC.
a) $5\sqrt{2}$
b) $10\sqrt{2}$
c) 10
d) 20
4. Using the cosine theorem, find the length of side AC in triangle ABC if AB = 8, BC = 10 and angle B = 60^0.
a) 8
b) 6
c) $5\sqrt{3}$
d) 10

5. A bisector AD is drawn in triangle ABC, dividing side BC into segments BD=3 and DC=4. Find the length of bisector AD if AB=5 and AC=6.
a) 4,5
b) 5
c) 6
d) 3,5

***Option* 2**
1. Find the radius of the circumcircle of a triangle if its sides are 7, 24 and 25.
a) 12,5
b) 14
c) 15
d) 16
2. In an isosceles triangle ABC, AB=AC=10, BC=12. Find the height from vertex A to base BC.
a) 8
b) 6
c) 7
d) 9
3. Find the radius of the inscribed circle of a triangle with sides a=8, b=15, c=17.
a) 3
b) 4
c) 5
d) 6
4. In triangle ABC, the angles A = 45^0 and B = 45^0. Find the angle C.
a) 90°
b) 60°
c) 45°
d) 30°
5. In triangle ABC, side AB = 10, angle A = 30^0, and angle B = 45^0. Find the side BC using the sine theorem.
a) 10√2
b) 15√3
c) 12√3
d) 5√6

Appendix C

An Example of a Student Survey

1. How interested are you in learning geometry using interactive methods and digital technologies?
☐ Very interesting
☐ Interesting
☐ Neutral
☐ Not interested
☐ Very uninteresting
2. Do you feel your geometry skills have improved thanks to the new methodology?
☐ Significantly improved
☐ Improved
☐ Not changed
☐ Deteriorated
☐ Deteriorated significantly
3. How easy has it become for you to understand and apply planimetric formulas and theorems in practice?
☐ Very easy
☐ Easy

☐ Neutral
☐ Difficult
☐ Very difficult

4. How would you rate your satisfaction with the learning process using the new methodology?
☐ Very satisfied
☐ Satisfied
☐ Neutral
☐ Dissatisfied
☐ Very dissatisfied

5. Would you like to continue your studies using similar interactive and digital methods?
☐ Yes, I would like to
☐ Yes
☐ Not sure)
☐ Rather not
☐ No

Appendix D

An Example of a Survey Among Teachers

1. How much do you think the new methodology has increased students' interest in geometry?
☐ Significantly increased
☐ Increased
☐ Did not change
☐ Reduced
☐ Significantly reduced

2. Do you see improvements in students' understanding of planimetric concepts and formulas?
☐ Yes, it has improved a lot
☐ Yes, it has improved
☐ Has not changed
☐ Worsened
☐ Deteriorated significantly

3. How much easier is it to explain complex geometric concepts using the new methodology?
☐ Much lighter
☐ Easier
☐ No change
☐ Heavier
☐ Much heavier

4. Assess the degree of student engagement in the learning process using interactive methods:
☐ Very high
☐ High
☐ Average
☐ Low
☐ Very low

5. Do you plan to continue using this methodology in the future?
☐ Yes, definitely
☐ Yes
☐ Possibly
☐ Rather not
☐ No