

Mathematical Learning Experiment Using Digital Technologies

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Abstract

The article examines the benefits of learning experiments using digital technologies in research based mathematics learning. We discuss the didactic purposes and tasks of learning experiments in mathematical education. In order to demonstrate the practical advantage of using digital tools, we describe four specific examples of mathematical learning experiments, based on Geogebra, Desmos and Maple.

Keywords

mathematical learning experiment, digital technologie, research-based learning, GeoGebra, Desmos, Maple

1. Introduction

Modern trends in the educational paradigm change are reflected in the Council of Europe document [1], which defines research-based learning as the leading educational methodology of the 21st century. The research study is possible only when students act as active study subjects and can solve cognitive problems. At the same time, mathematical experiments, observations and research are some methods of implementing such an approach to learning. They enable the involvement of students in active search activities and the implementation of research-based learning. The result is not only the solution to educational problems but also the formation of students' motivation to study mathematics, cognitive activity, the ability to work independently and in collaboration, etc. Using digital technologies for conducting experiments contributes to a better understanding of mathematical concepts, the active student's involvement in constructing their knowledge, and developing their research skills.

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2. Literature Review

To date, several studies on the problems of experimental mathematics have been implemented. These studies revealed the possibilities of using its various methods, particularly using digital technologies [2, 3, 4, 5, 6]. The previous research of this study's authors shows the advantages of digital visualisation in forming students' mathematical competence. In addition, it disclosed possibilities of involvement of digital visualisation tools for conducting mathematical experiments using dynamic geometry environments GeoGebra and advanced graphing calculator Desmos [7].

However, despite numerous studies, the possibilities of learning experiments in teaching and learning mathematics and using digital tools for its implementation still need to be presented.

Therefore, the article aimed to demonstrate the possibilities of learning experiments using digital tools for research-based mathematics learning.

3. Research results

3.1. Aims and stages of mathematical learning experiment

The mathematical experiment's aim in learning mathematics is to form the student's ability to think mathematically: to experiment, observe, notice patterns, check theoretical results, etc. That is, to give students a full-fledged mathematical experience. A mathematical experiment is used to formulate hypotheses based on observed facts and properties; to check the hypotheses or the correctness of the results established analytically; to visualize mathematical concepts, facts, and principles for a better understanding of them; to predict a possible result to assess whether it is worth a formal proof; to see the result immediately, if its rigorous mathematical proof is not the ultimate goal; to replace (automate) cumbersome manual calculations and complex transformations.

In [2], it has been proven that learning mathematics aims easier to implement if they are supported by an experiment, particularly with the involvement of digital technologies.

We highlight the stages of a mathematical learning experiment using digital technologies are as follows.

1. Determination of the experiment learning aim, formulation of its task as a mathematical research problem, justification of the feasibility of involving digital technologies, and selection of digital tools.
2. Solving the research task: developing a computer model, conducting a mathematical experiment, obtaining its results (formulating a new mathematical hypothesis, setting a problem, developing a certain algorithm, refuting the result obtained analytically).
3. Reflection and conclusions regarding achieving the research and learning aims.

3.2. Examples

Example 1. The mathematical experiment as an element of research-based learning involves changing the key parameters of the built model to observe its behaviour. GeoGebra makes building dynamic models of mathematical structures and geometric objects possible [8]. By creating and researching such dynamic models, the student can observe the properties of geometric objects that are invariant under various transformations. For example, let's consider the mathematical experiment in GeoGebra when studying "Pascal's theorem in a projective plane".

The student is invited to investigate the collinearity of the three points of intersection of the opposite sides of a hexagon inscribed in a conic (it may be an ellipse, parabola or hyperbola). The simplest image is a convex hexagon inscribed in an ellipse (Fig. 1a). By moving one of the vertices of the hexagon along the conic, the student can observe other configurations: the hexagon with self-intersections (Fig. 1b); another conic (if you move one of the five vertices-points defining the conic, Fig. 1c). But with any transformations, the points of intersection of the opposite sides of the hexagon are on the same straight line (Fig. 1a–c). Thus, the student can summarize the results of the experiment in Pascal's theorem: if six arbitrary points are chosen on the conic and joined by line segments in any order to form a hexagon, then the points of intersection of the pairs of its opposite sides are collinear (lie on a straight line — Pascal line).

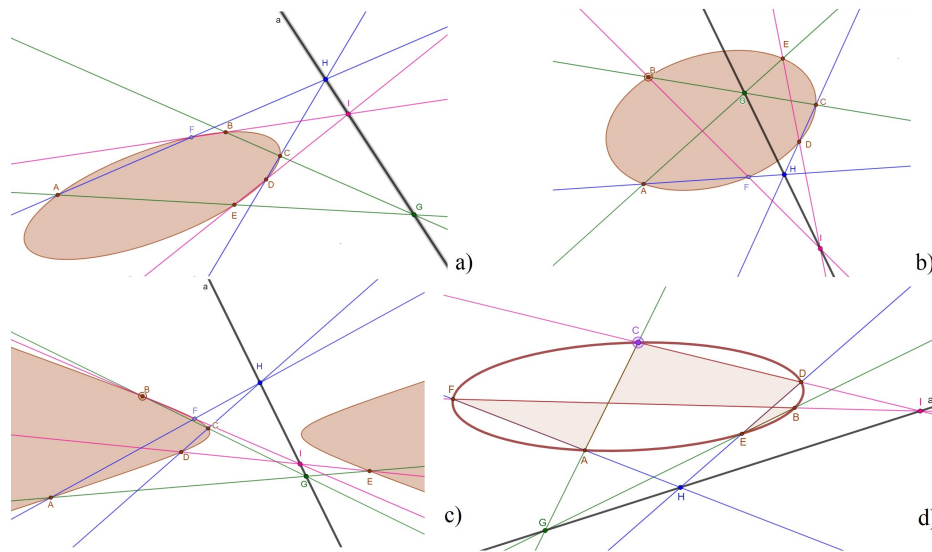


Figure 1: The dynamic model to Pascal's theorem

If students construct an arbitrary hexagon $ABCDEF$; find the points of intersection of three pairs of opposite sides of the hexagon; draw the straight line through two of the opposite sides (for example, G and I); by moving the vertices of the hexagon, ensure that the third point of intersection of the opposite sides (point H) falls on the GI line (Fig. 1d). Then, they can find that the sixth vertex of the hexagon (point C in the figure) falls

on the conic defined by the other five vertices. That is, we have a statement that is the inverse of Pascal's theorem.

Example 2. One of the concepts of mathematical analysis that causes first-year mathematics students the most difficulty in understanding and mastering is the uniform continuity of the function. The formal definition of the uniform continuity of the function of one variable looks like this:

The function $f(x)$ is called uniformly continuous on the interval X if for any $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon) > 0$ such that from the condition $|x_1 - x_2| < \delta$, the inequality $|f(x_1) - f(x_2)| < \varepsilon$ follows for any x_1 and x_2 from the set X .

A lack of understanding of the essence of the concept leads to the fact that many first-year students cannot correctly reproduce this definition, or, having invented it, they quickly forget it. Therefore, we begin the study of the concept of the uniform continuity of the function with a visual experiment using GeoGebra. Let's describe this experiment.

If the function $f(x)$ is continuous on the interval X , then it is continuous at any point x_0 of this interval. That is, $\forall \varepsilon > 0 \exists \delta > 0 : |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$.

It is evident that δ depends not only on ε (the smaller ε , the smaller δ) but also on the point x_0 . If the value of δ does not rely on the point x_0 , that is, the same δ is suitable for all points of the interval X , then the function is uniformly continuous on this interval.

Consider in GeoGebra the graph of the function $f(x) = 1.5 \log_{0.5}(x + 0.5)$ on the interval $(-0.5; 2]$. Construct a rectangular frame with height ε and width δ . We move this frame along the graph of the specified function in the direction from right to left. Let's try to make it so that the curve crosses only the vertical sides of the rectangular frame so that the values of the function differ less than by ε . By moving point x along the abscissa axis from right to left, students make sure that at some point, the curve "breaks out" from a rectangular frame on its upper side (Fig. 2a).

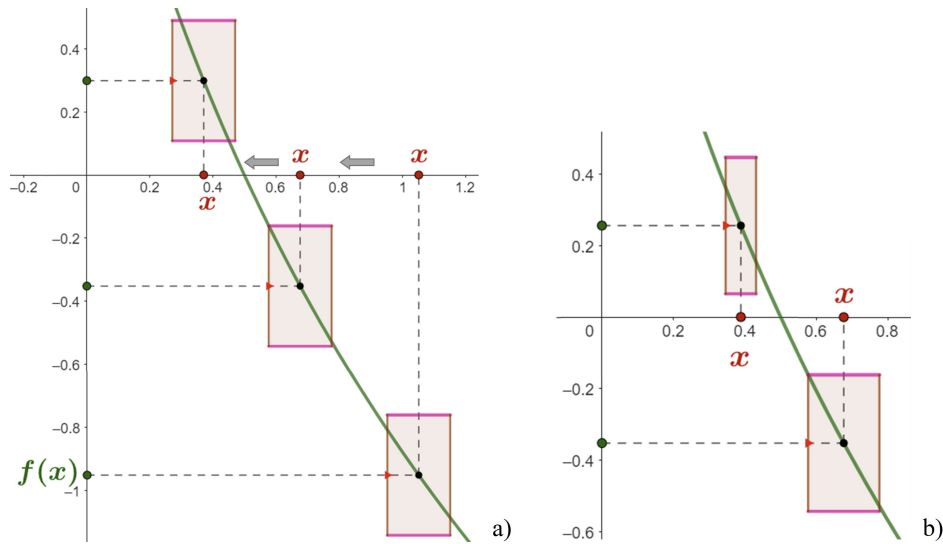


Figure 2: The dynamic model to definition of the uniform continuity of the function

This means that it is possible to specify two argument values that differ by less than δ , and the distance between their corresponding function values is greater than ε . To “keep” the points of the graph within the height of the frame, this frame will have to be narrower, i.e. reduce δ (Fig. 2b). You can see that the new frame will not satisfy us for long. Designing increasingly narrow frames will not help because one frame cannot cover the entire infinite curve, no matter how narrow it is. That is, this function is not uniformly continuous on the interval $(-0.5; 2]$.

However, students notice that if we consider the same function on the interval, for instance, $[-0.49; 1]$, i.e. cut it off, it is possible to find a frame with which to “pass” the entire curve. Thus, they reach a hypothesis about the uniform continuity of the function on a closed interval (Cantor’s theorem, one of the most important theorems of mathematical analysis).

Finally, we note that preparing the described dynamic model (as well as many others) is difficult. Therefore, we often involve senior students in the development of such models. This is a valuable practice for them.

Example 3. The task of calculating the limit

$$\lim_{x \rightarrow \infty} \left(x - x \cos \frac{1}{x} \right)$$

caused difficulties for first-year mathematics students. One of the students suggested using a graphic representation of a function whose limit at infinity should be found (Fig. 3). This made it possible to see the result (0), after which an analytical solution method was found.

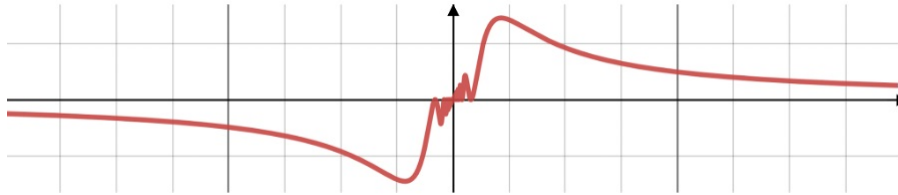


Figure 3: The graph of the function made in the graphic calculator Desmos [9]

If finding the boundary was not the main task but only an intermediate stage of solving some problem, the result obtained based on a graphic image would be reasonably sufficient.

Example 4. Maple has a wide range of possibilities for conducting educational mathematical experiments in higher mathematics [10]. The example of the Maple application is studying the equilibrium points types of differential equations systems. By changing the system’s parameters, the student can visually identify equilibrium points and analyze the movement of system solutions around these points. By changing the coefficients of the Lorentz system of differential equations, the student analyzes the changes in the oscillogram graphs (Fig. 4a) and the appearance of the Lorentz attractor projections on the planes of the three-dimensional phase space (Fig. 4b) based on the solutions in Maple. In this case, these projections are irregular orbits around unstable fixed points.

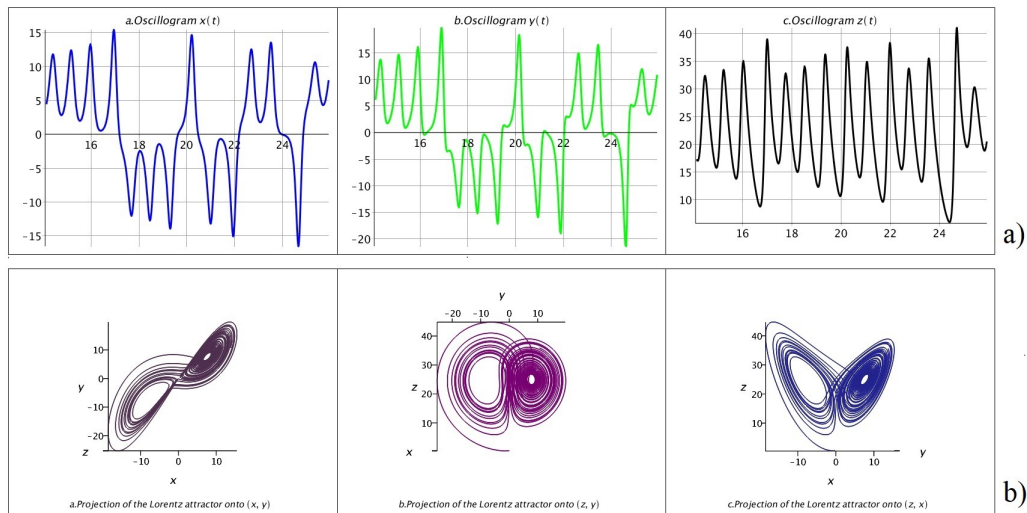


Figure 4: Investigation of the equilibrium points of differential equations systems

4. Conclusions

It has been demonstrated that learning experiments, as a teaching and learning strategy, have an essential role in the mathematical preparation of students, forming their full-fledged mathematical experience. Digital technologies, in turn, provide a wide range of interactive opportunities for experimentation and exploration.

The possibilities of learning experiments using digital tools for research-based mathematics learning are illustrated by specific examples.

Analysis of the effectiveness of learning experiments using digital technologies in teaching and learning mathematics will be the subject of our further research.

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