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APPLICATION OF INTERACTIVE EDUCATIONAL METHODOLOGIES IN FORMING MATHEMATICAL THINKING IN COLLEGE STUDENTS

Abstract. *In this paper the notion of the essence of mathematical thinking as a mental process is analyzed, the relationship between studying mathematics and thinking development in enlightened; the factors affecting the search of conditions of effective mathematical thinking development are clarified; Organizational forms, methods and techniques of teaching of innovative nature are considered.*

Key words. *Thinking, mathematical thinking, types of mathematical thinking, interactive education, interactive education techniques.*

Introduction.

One of the tasks of teaching mathematics at the higher education is insuring the level of mathematical culture enabling students to carry out their future professional duties to their full extent. Mathematics is a unique tool of developing not only educational, but also intellectual potential of any individual. Mathematics, as a science, combines general and abstract knowledge, and is used in all areas of expertise. The main goal of mathematics is giving a description of certain process, i.e. in the language of formal logic. Speaking about the subject and the functions of mathematics, it is obvious that the integrating role of mathematics in the modern science is becoming increasingly essential, since it is generically a scientific discipline. At the same time functions of mathematics are humanitarian since they are àiming at an improvement of material and spiritual spheres of human existence. Students' intellectual abilities get developed during studying of mathematics and other mathematically relative disciplines. Teaching of any area of mathematics beneficially affects students' intellectual development since it grafts their skills of clear logical thinking, that operates on clearly defined notions. Engagement of interactive technologies promotes better understanding of sophisticated mathematical material and develops students' thinking.

Therefore the **main aim** of this paper is defining the essence of mathematical thinking as a mental process and its components; establishing interrelations between teaching mathematics to students and developing their thinking abilities. It becomes possible with the aid of the following **tasks**: identifying the factors that affect conditions of

students' effective mathematical thinking development; finding organizational forms, methods, techniques of education that provide maximum extent of developmental function of mathematics; justifying the necessity of applying interactive technologies.

The Main part

Instructors of mathematical disciplines are facing the problem of forming and developing of mathematical thinking of future specialists, i.e. theoretical thinking that is built on mathematical objects. While doing that it is necessary to identify the essence of mathematical thinking and establish interrelation between teaching mathematics to students and developing of their thinking. This will help to find such forms of education, methods and techniques, in particular application of interactive ones, that can make the developmental function of mathematics transpired.

Thinking is a process of mediated and generalized reflection of objects and phenomena of objective reality in their essential connections and relations by a person. Human thinking is inextricably related to speech, which is a tool of forming and expressing a thought. Human thinking is also naturally related to practice. Practice is a source of any thinking activity [5, p. 280]. The following are the tools of thinking: analysis, synthesis, comparison, abstraction, generalization, classification, systematization. The following are the forms of mental reflection: judgments, considerations, conclusions, concepts. The following parameters are individual features of thinking: independence, criticism, flexibility, depth, breadth, consistency, speed, rigidness. In the educational activity thinking is a process of acquiring knowledge, that is built-up in accordance with a psychological *knowledge acquiring theory*, that conditionally contains the following stages: perception (based on sensors), comprehension, generalization, practical actions. The process of gathering educational knowledge occurs on the base of knowledge gathering methods – verbal, visual, practical. If it is necessary to make this process more sophisticated, for example, the process of perception and comprehension is built upon a more complicated methodology of problem-based, independent study, then, in such a case, the intellectual activity to the high extend stimulates the final stage – abstract thinking (generalization). As a rule, when one is talking about intellectual development in the process of studying mathematics, he or she bears in mind the development of mathematical thinking. This, indeed, is correct: in the process of teaching mathematics the major concern is specifically development of mathematical thinking, not just thinking in general.

The prominent soviet mathematician A. Khinchin, one of the most outstanding scientists in the soviet probability theory school, who had a deep interest in the mathematical education problems, has pointed out *four specific attributes of mathematical thinking*: 1) an irreproachable logical schema of thoughts; 2) conciseness, conscious aspiration to find the shortest logical path to the target; a merciless rejection of anything that is not absolutely necessary for a concise argument; 3) a clear passage of arguments; 4) meticulous accuracy of notation [3, p. 38].

Studying mathematical disciplines in higher school is a complicated process, that has the following aiming components: adopting of mathematical knowledge by the students; 2) mastering certain mathematical skills and techniques by the students; 3) developing thinking in students. Until recently it was believed that the successful achievement of the first two objectives automatically assumes a the successful achievement of the third one, i.e. it was believed that development of mathematical thinking occurs spontaneously in the studying process. This is true, but only to a certain extent. The results of many domestic and foreign psychologists and pedagogies show that mathematical thinking is not only the most important component of knowledge-seeking process, but also is such a component that, if its ambitious development is absent, then no efficient results in mastering mathematical science are possible.

Summing up the, mentioned above, under *mathematical thinking* we understand a combination of intellectual individual peculiarities of a human being that are involved in the process of gaining mathematical knowledge or in the process of applying mathematics in different areas of science or industry.

These peculiarities of thinking, that are present in the very nature of mathematics, are connected to the objects studied, as well as to the methods used in these studies. There exists a general idea concerning active work of certain individual thinking peculiarities during the process of mathematical thinking. (flexibility, consistency, speed, criticism, economy, depth, breadth, independence). Taking into account the importance of all individual thinking peculiarities we want to point out those of them that are the most important for mathematical thinking. In our opinion that is a «thinking kernel» – flexibility, depth, consistency, speed.

The flexibility of thinking is a skill of purposefully changing a method of solving a cognitive problem, easing a transition from one way of doing that to another, an ability of breaking a habitual sort of actions, finding new ways of solving the problem while given conditions (parameters) are changing. The highest level of the development of irregular way of thinking is manifested in originality of thinking, which in the process of studying mathematics acts as ingenuity of methods used for solving problems, familiar to the students. The flexibility of thinking is manifested in the readiness of quickly switching from one method of solving a problem to another, changing tactics and strategies, finding new non-standard ways of acting under conditions that change.

The depth of thinking is characterized by the skills of grasping the essence of every fact being studied in their mutual relations with other facts, identifying hidden peculiarities in the material, seeing the problem where others do not see it, foreseeing possible outcomes of events and processes.

The consistency of thinking manifests in a skill of following logical sequence of ex study pressing judgments and their justification. We say that thinking of a specific individual is consistent if he or she closely follows the topic of reflections, neither diverts from it nor jumps from one topic to another nor substitutes the subject of reflection. For a consistent thinking it is typical to follow certain principles of considering a

question and a plan of comprehension, absence of contradictions and logical mistakes in thoughts justifications, provableness and objectiveness in conclusions being made.

The speed of thinking is the ability to quickly sort out a complicated situation, quickly find a correct solution and adopt it. The speed of thinking depends on the knowledge, the level of formation of thinking skills, the experience in the corresponding area and the mobility of processes in the central nervous system [5, p.292].

That is why mathematical thinking is abstract theoretical thinking, objects of which are immaterial and can be interpreted in any arbitrary way under the condition of preserving given relationship among them.

Some psychologist believe that in the general thinking structure one can allocate 5 subsets – types of mathematical thinking. This classification looks as follows: 1) topological thinking; 2) serial thinking; 3) metric thinking; 4) algebraic thinking; 5) projective thinking.

Clearly all these types of thinking are present in every individual. By the way, the serial thinking dominates in the majority of people. That can be explained by the fact that the lower school educational system is serial. The dominant defines many aspects of intellectual, and subsequently practical activities, however, not only in the area of mathematics.

Scientists have found out that individuals of the same type of thinking feel mutual attraction since it is difficult for them to understand people who are «mathematically different». Hence, it is important to develop all types of thinking in students. According to the current domestic periodization the youth age embraces two consequent phases, where the age from 15 to 18 is defined as the early youth, and the youth itself lasts from 18 to 21. College students depending on the year in college belong to one of these youth phases. Studying age psychology we can observe that development of cognitive sphere in early youth reaches the certain level when the individual is ready to carry out practically all kinds of adult intellectual activity. A number of the scientists (J.Piaget, Ya.Ponomaryov) all together point out the peak of achievements of intellectual strength at the end of this period. For the given age period the driving activity is the study-professional one, i.e. the study leading to mastering a profession. Since college students are professionally oriented already, a new social position and leading activity are changing the relevance of the scolar process, in particular, their study for them. The interest in studying is increasing in them comparing to secondary school students. It is related to forming a new motivational structure of their study. Motives, related to their future profession, and preparation to self-reliant life style become dominating. In college students a conscious positive attitude to the studies manifests itself. They are interested in the subjects, that will be required in their life ahead. They are concerned about their success of the study. Since mathematics is one of the significant subjects, that is applied in many disciplines, studying of mathematics is particularly important for the students.

Development of mathematical thinking with the aid of interactive technologies.

As L. Khoruzha is pointing out «Establishing and development of a human being today proceeds according to a different scenario than it did before. An individual differently perceives and digests the information» [4, p.16]. That is why in our opinion, the application of the interactive education in the process of forming mathematical thinking of future elementary school teachers is extremely important. While becoming a subject of education rather than an object during the educational process, feels a student the need of being an active participant of the action, as well as his own education and development. This insures the internal motivation of studies, which promotes its effectiveness. As a scientist V. Khiminets is pointing out: «Under interactive technology of education one should understand such organization of educational process, at which the studying person necessarily takes part in a collective mutually supplementing, based on interactions of all its participants, educational process of comprehension. Every participant of the interactive education has a specific task that is reported publically. The quality of this report impacts the work of the whole group he belongs to. Interactive technologies of this education by their essential contents and the structure assume concisely planned educational result [2, p.257].

Interactive technologies of education allow the development of student's individuality, his thinking, including the mathematical one, taking into account the individual peculiarities. Due to implementation of interactive technologies under certain education conditions for the effective educational environment, the development, self-development and guidance are stimulated. During an interactive session the student has an opportunity to express his personal attitude to the material, exchange knowledge, ideas, thoughts, ways of acting, so the complex acting of the following «thinking kernel» gets manifested: flexibility, consistency, speed of thinking. The goal of an educational session is planned considering joint activity of instructor and students, taking into account student's needs, involving the subjective experience of each of them. This would characterize the education as a genuine personally oriented process. The individuality of every student, his unique, unrepeatable peculiarity is taken into account. During this interactive education it is impossible for one person to dominate over another, and one student cannot dominate over another student. Comfortable educational environment gets created, every student feels his intellectual capability, the atmosphere of mutual emotional and intellectual support as well as positive friendly mutual dependency gets organized. Interactive technologies enable to focus educational process on the students' individual development and self-advancement. These technologies give an opportunity to combine individual, paired, grouped and collective kindest work. Their implementation is conditioned by the modeling real life situations via stimulation and simulation games, also via resolving problematic situations, conducting discussions, etc.

Therefore the signs of interactive education are: an existence of a common goal (however not identical for all the students), and clearly planned and expected result of education; aspiration to involve a subjective experience of every student during the

educational process; education is built on the basis of a dialogue, for example between an instructor and students, or among the students; positive mutual dependency of the students, creativity, collaboration during the education process; achieving personal success is possible only under a condition of the success of all the participants of the educational process; activity, leadership of all of students during the educational process; creating comfortable educational environment in such a way that students can feel their intellectual ability; existence of a challenging assignment, exchange of knowledge, ideas, methods of activity, etc.; a personal point of view is expressed and justified (or gets changed due to an argument) in the atmosphere of mutual support and affability; a domination of one thought over another or one opponent over another becomes impossible; combining individual, paired, grouped and collective work.

The following methods characterizes the interactive education: a microphone, a brain storm, taking a position, learning while teaching, working in paires, working in triples, acting out story based problems, a delicate saw, a circle of ideas, an aquarium etc. [6, pp. 34-45].

Let us consider some of them. We will bring to your attention fragments of a class on the topic «Area of curvilinear trapezoid. Definite integral. Newton-Leibniz formula». This class was taught by the author to the sophomore students with the use of interactive education technologies as a part of a general course of mathematics in a creative laboratory of the cyclic commission of economics-mathematical disciplines and management of the University College of Borys Grinchenko Kyiv University.

A class objective are: to familiarize students with problems leading to an integral concept, in particular with the problem of the area of a curvilinear trapezoid; to form the skill of computing area of the regions bounded by curves.

The author has been working with the students for more than a year, and knowing capabilities of every student she formed a group of the most capable students («home group»), gave them homework assignment to prepare some part of the material that would be taught at the next lecture, aiming to apply interactive technology of studies and to make an educational process more realizable and interesting.

In order to hint students at the topic formulation, the slide with the images of a rectangle, triangle, curvilinear trapezoid in Cartesian coordinate system was prepared. The two first figures and formulae for their area are familiar to the students from the previous courses, but the last figure and the formula of its area are currently unfamiliar to the students. Applying L.Vigodsky's theory about zones of closest development, the instructor stimulated interest to the educational process steering students to the solution of the problem that currently are beyond the limits of individual abilities, but with tescher's and «home group» students help they get necessary knowledge.

The interactive «Microphone» method was used for a discussing and understanding the area of displayed figures. This method enables everyone to express his thought and ideas quickly, in turn, answering a question.

- What figure is displayed in the first picture? How to calculate its area?
- What figure is displayed on the second picture? How to calculate its area?

The rules of conducting a «Microphone» are: 1) only the student holding the «Microphone» can speak (a student names the figure, the formula for calculating its area, and orally computes the area); 2) answers are not commented, nor evaluated; 3) when one students speaks, the others keep silent.

Having discussed familiar figures, a rectangle and a triangle, the question how to calculate the area of a figure bounded by an arbitrary curve arises. After that the instructor gives a definition of curvilinear trapezoid, and hints how to compute the area by using the method «learning-while- teaching». This method enables students to take part in the educational process, conveying knowledge. A student from the «home group» has prepared the proof of the theorem concerning the area of a curvilinear trapezoid.

Rules of applying the method are: 1) instructor specifies the topic and the objective of the session, and gives out cards with assignments (distributes some part of the lecture to every member of the «home group», different assignment for each student); students familiarize themselves with the information; 2) if a student is confused with something he may clarify it with the instructor; 3) students prepare to convey information to other students in an easy way; 4) the task is to exchange the information with other students in the group; 5) after the exchange is completed, students share with the group what new they have learnt and done.

After the student has proved the theorem about the area of curvilinear trapezoid, the instructor generalizes the idea up to a definite integral and Newton-Leibniz formula, the «Brain storm» has started, which is an effective method of collective discussion, search for a solution, that stimulates participants to demonstrate their imagination and creativity, develops mathematical thinking. This method includes free expression of thoughts by all participants and aids in finding many ideas and solutions.

Rules of conducting the «Brain storm» are: 1) all participants of «the storm» offer ideas concerning the solution of the problem under consideration (how to apply new formulae for computing the area of curvilinear trapezoid defined by curves, the condition is displayed on a slide); 2) one of the students («the secretary») writes down all the suggested ideas on the blackboard; 3) the ideas are grouped-up, analyzed, discussed in a group, (it is possible to improve somebody else's ideas); 4) choosing the best solutions [7, p.190].

The code of behavior during the «Brain storm» is: trying to propose offer as many ideas for solving the problem as possible; turning-on your imagination: do not turn down any idea for the only reason it contradicts the common thought; one can offer any number of ideas or expand the ideas of other participants; criticism or evaluation of somebody else's ideas are not allowed.

In order to gain practical skills in evaluating integrals and computing the area of curvilinear trapezoid « the work in pairs» follows up. This form of work allows students to gain collaboration experience, master the skills, express themselves and actively listen.

Rules of conduct are: 1) students read the assignment and pertaining information before it is conducted; 2) it is established who will speak first; 3) any thoughts concerning, the perception of the problem are expressed in turns; 4) the common thought

must be worked out; 5) it is decided who will present the results to the entire group, and then necessary preparations for this are made.

Summarizing such a session it must be said that the implementation of the interactive technologies is very hard and tedious work of the instructor. He must be capable to organize such work, and has to have all necessary knowledge. Computer technologies especially an interactive blackboard, that currently is becoming very popular and exists in many educational institutions, essentially supports the instructor's work.

Conclusion:

Interactive technologies in education used in mathematical classes contribute to an effective development of mathematical abilities in every individual, their «thinking kernel» – flexibility, depth, consistency and speed; development of human values, generally accepted norms of behavior during mathematical lessons, as well as in casual life; development of an ability to cherish knowledge and skills to use it; acknowledgement of personal responsibility, development of skills to unite with other members of a group for solving a common problem, development of an ability to recognize and respect somebody else's values, forming social skills, and collaborations with other members of a group, mutual understanding, and respect of every individual, forming a tolerance, sympathy, goodwill and care, solidarity and equality, forming skills to make a free, independent choice, that is based on the own judgment and analysis of reality, understanding of norms and rules of behavior.

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